

STOCHASTIC MODELLING OF THE MICROGEOMETRICAL
STATE OF A MANUFACTURED SURFACE

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ABSTRACT

This thesis presents a method of analyzing surface irregularities through an analogy to stochastic processes. The stochastic functions such as the mean value, the mean square value, the probability density and distribution, the autocorrelation and the spectral density of a manufactured surface are defined and represented in a form suitable for computer analysis based on discrete measurements. A comprehensive computer program is developed to accommodate 4,000 data points. Experiments were conducted on surface samples obtained through a grinding process, employing different sampling rates and sample lengths. Results are presented both in discrete form as well as in continuous plots, and are discussed with regard to surface characterization. Finally, the spectral density is considered the best single function descriptor of the surface texture since it provides information in the amplitude domain, as well as in the length-wise domain.

NOMENCLATURE

The following represent symbols and notations utilized in this work. Some of these symbols are also defined as they appear for the first time in the text.

A	upper limit of the amplitude domain
B	lower limit of the amplitude domain
C	correlation length
c	size of the amplitude class interval
f	frequency
f_0, f_c	Nyquist frequency
G	the raw spectral density estimate; (Glattungstiefe)
h	sampling time
i, j, n	indices
k	number of amplitude class intervals; harmonic number
l	independent length variable
L	length of the sample surface, mm
m	maximum autocorrelation lag number
N	length of the sample surface, (as number of data points)
p	probability density function
P	probability distribution function
R_{\max}	maximum peak-to-valley amplitude
R	autocorrelation function
S	spectral density function

T_a	bearing area;(Tragenteil)
T	fundamental period
$U(\ell)$	surface texture analog function
U_n	surface texture discrete function
v	speed of the travelling stylus of Talysurf 4
$Y(\ell)$	normalized surface texture-analog function
Y_n	normalized surface texture-discrete function
$Z(\ell)$	normalized and standardized-analog function
Z_n	normalized and standardized-discrete function
$\bar{U}, \bar{Y}, \bar{Z}$	mean values of the functions U, Y and Z
$\bar{U}^2, \bar{Y}^2, \bar{Z}^2$	mean square values
$\{ \}$	random process
$< >$	ensemble averages
τ, r	autocorrelation (length) lag
ϵ	statistical error of the spectral density
λ	wave length; distance between sampled points
ρ	autocovariance
σ	variance
$\#$	number

CHAPTER I
INTRODUCTION

CHAPTER I

INTRODUCTION

1.1 Preliminaries

The importance of the surface texture irregularities in engineering is historically related to a number of practical observations associated with the functional behaviour of shafts and axles in the automotive and railway industries. The first observation emphasises the resistance of the surface to the corrosive effect of the atmosphere. It was observed that out of a typical lot of 100 parts, stored under similar environmental conditions, (smoke, humidity and temperature), the parts having a "smoother" surface texture provided a better resistance to corrosion than those characterized by a "rough" surface texture. Since then, many experiments on this subject have verified the above basic observations.

Another observation on the effect of surface texture on manufactured products emerged from the premature failure of certain specimens in a group of shafts and axles. A detailed comparative analysis of the region of failure of the shafts indicated that a loss of fatigue strength was noticeable where the surface was "rough". Apparently, this introduced a relationship between the texture of the manufactured product and its ability to withstand cyclic loads. These observations resulted in the introduction of certain surface texture specifications in manufacturing. Often the specifications were introduced by selecting a particular product

with acceptable surface smoothness for a continuous comparison. Even though smooth surfaces assisted in improving the fatigue and bearing strength of the product, it does not follow that they are always desirable. Experiments have shown that for an optimum lubricability some roughness is necessary as it helps retain the oil in the valleys of the surface irregularities. Some "controlled" roughness will therefore improve the life of the bearings and surfaces in contact. It may be noted from the above statements that one specific surface texture, to suit all the expected functions of the manufactured product, does not exist. Maximum fatigue strength and maximum lubricability cannot be achieved simultaneously and a compromise should be looked for. It should also be borne in mind that overfinishing, even though desirable, is uneconomical.

It may then be concluded that a quantitative description of surface texture is necessary to achieve proper functional behaviour of the product and must be introduced in terms of specifications in all manufacturing processes.

1.2 A Critical Review of the Existing Methods

Many measuring methods have been developed to give a numerical characterization for the surface irregularities. These attempts have resulted in the definition of a number of national and international standards^{[1] - [6]}, all of whom recommend the surface texture characterization based essentially on a single-valued parameter. These parameters can be

computed with relatively simple algorithms. Some of the commonly known methods falling into such categories are the peak-to-valley method, the CLA method, the Swedish method, and the British 10 point criteria.

Because of the complexity of the microgeometry of a manufactured surface, and the random characteristics of the heights and valleys of the irregularities, a precise definition through the use of a single index is almost meaningless. For this reason, the standards presently available and the definitions which are associated with them are not completely satisfactory. They do not represent the outcome of a sound and logical mathematical model of the surface, but rather an accumulation of definition and procedures that have been developed somewhat arbitrarily to satisfy specific rather than general needs.

The measuring methods indicated above can be classified into the following categories, namely, empirical, semi-empirical and statistically based criteria. The British 10-point measure or the peak-to-valley measure falls into the first category; the CLA-value may be best incorporated into the second one. In the third category, one records the bearing area and the rms value which is actually out of the circuit usage, since it has been replaced by the CLA-value. The usage of the existing methods poses a number of difficulties from a practical point of view. Some of the inconveniences are the following:

- (i) The indices which are produced by using any of the existing national standards do not give a true representation of the surface texture. There exists a number of cases, Rupert [7], of different surface textures that share the same numerical assessment. Another difficulty is due to the fact that the existing surface-indices do not bear any exact relationship to the total description of the surface.
- (ii) Some of the existing methods are intended for immediate utility and therefore should be regarded as shop-criteria. They are subject to manual calculations and personal interpretation which tend to reduce their objectiveness. For instance, the maximum peak, or the maximum valley, the British 10 point system, are of this type. Other methods such as the CLA or the rms-value have gained a larger recognition, particularly since these parameters lend themselves to be incorporated into direct measuring devices.
- (iii) A third and important difficulty is associated with the problem of technical communication, i.e., the problem of translating the engineering drawings from a particular surface criteria, say the CLA-value, to another particular criteria, say the British 10-point method. The problem of obtain-

ing equivalent values for these parameters corresponding to the different measuring methods, is still not clear and there actually exists only a limited number of such conversion factors in current usage.

These difficulties can be overcome only if uniform surface texture specifications, based on the statistical characterization of the real surface itself, are proposed. Such a description will involve a probabilistic approach to the modelling of surfaces using the mathematical theory of random processes.

1.3 The Probabilistic Characterization of the Surface Geometry

In the practice of surface texture analysis, a number of measuring techniques have been developed for producing a reasonable magnification of the microgeometry of the irregularities. Careful and extensive analysis of a large number of such profiles, obtained by mechanical or electro-mechanical devices, have clearly indicated that the surface texture is basically in the form of a random signal.

A complete characterization of the surface texture as a random process is analytically possible by generating an infinite number of moment functions, if the probability density functions are known. However, from a practical point of view, the following selected statistical functions can produce satisfactory surface texture characterization. They

are: the probability distribution, the autocorrelation and the spectral density functions. Usually, the specification of the surface texture, by using some or all of these descriptions, result in a functional vector of a tabular form or as a series of plots of the appropriate functions.

Nakamura [8] and Onishy [9] utilized the autocorrelation and the dispersion spectra, or the root-mean-square value, for describing the surface texture. The concept of the amplitude density is introduced by Pesante [10], where he considers the surface as a randomly varying signal and attempts to describe it by using the probability density and the probability distribution functions. The first conclusion is that very few samples present a Gaussian distribution, and secondly, that for surfaces produced by same manufacturing process, the shapes of the probability functions seem to be uncorrelated to each other. This shows that the first order probability descriptions do not generate adequate surface texture characterization. Later, Myers [11] proposed additional statistical description. The new characteristics are the first and the second derivatives of the surface profile and their ratios. These were introduced to characterize the directional properties of surface signal measured. Peklenik [12],[13],[14] defines the roughness in terms of new parameters derived from the autocorrelation function of the random profile. In [12] Peklenik introduced the correlation length and the wavelength correlation. These two parameters are meant to produce a classification of the surface texture

profiles into one of the following five categories:

- (i) A deterministic profile;
- (ii) A wide band process;
- (iii) A deterministic function and a super-imposed random wave, the autocorrelation being obtained by making the summation of the two corresponding autocorrelelograms; it does not decay to zero for any correlation length.
- (iv) A carrier profile and a superimposed random function, as the most common case, and
- (v) A surface texture that cannot be simply expressed in terms of the above-mentioned elementary functions.

Experimentation has produced for the correlation length values between 0.05 and 2.5 mm and for the correlation wave-length values in the range from 0 to 10 mm. If, however, several carrier frequencies are detected in the surface texture, the one with the maximum or largest correlation is selected as the fundamental one for the determination of the wave-length correlation.

Peklenik [13] also introduced the mean thickness of the profile and the autocorrelation of the slope. These two parameters can be computed in terms of the average number of crossings of the record, per unit length. The average profile thickness is defined as the ratio between the length

of the surface texture record above a preselected amplitude level "a", and the number of crossings at this level. The second parameter, the standard deviation of the slope, is computed by considering the second derivative of the autocorrelation corresponding to a zero lag value. This parameter helps to distinguish between sample surfaces that bear the same numerical values for their corresponding standard deviations. The variance of the slope reaches its minimum value at the level corresponding to the average-mean value and increases towards the peaks and towards the valleys. The concept of three-dimensional analysis is further introduced by Peklenik in [14]. For this, the autocorrelation and the cross-correlation are utilized. It is recommended that this kind of analysis be utilized in conjunction with previously defined surface texture parameters, as per [12] and [13].

Wallach [15] proposes a three-dimensional analysis of the surface texture irregularities by recording profiles along parallel traces and utilizes numerical analysis for the description. The regular waviness of the surface is indicated by the coefficients of a double Fourier expansion. The additive random roughness is described by the particular probability distribution. The results represent the description of the surface, independent of the manufacturing process.

Another unified approach to surface metrology is presented by Spragg and Whitehouse [16]. The authors supple-

ment the peak-to-valley measure, or the arithmetic average R_a , by an additional parameter, the average wave-length, to take into account the degree of openness or closeness of the surface texture. The peak-to-valley distribution is also introduced for the characterization of the asymmetry of the wave-form about the mean. An interesting approach is based on the modelling of the power spectrum as a first or as a second-order linear system, to compute the rate of variation of the average wave-length and of the average mean square angular frequency. The cut-off is also presented in relation to experimental results. Finally, for surface textures characterized by important asymmetries of the wave-form, a third parameter derived from the peak and the valley distribution is introduced. It is essentially a measure of the asymmetry of the profile about the mean line and consequently, a measure of the skew of the surface texture.

Sankar and Osman [21] have recently introduced the technique of random excursions for the surface texture characterization. The mathematical description of the probability density of the crest width and of the valley spacing is given. These probability functions, along with the average number of crossings of the surface function about the CLA level, provide the statistical definition of the surface irregularities in both the longitudinal and the transversal directions.

All the probabilistic analysis mentioned above use a

combination of different statistical descriptions for specifying the surface random process. The amplitude density function can be established as the primary descriptor and along with this one or more higher order statistical function may be used. It is not clear which combination provides the best description for machine shop specifications. One of the requirements should be that the specifications, even though through the theory of stochastic processes, must be simple enough for adoption by designers and machinists. The correlation function and the spectral density are the two relatively simple stochastic descriptors that can be used along with the amplitude density for convenient characterization of surface structures. Many previous papers, Nakamura [8], Peklenik [12] and [13], Spragg and Whitehouse [16] have concentrated in using the correlation function, since these discriminate between the periodic and the random components of the profile effectively. In this thesis, it is attempted to show that along with the correlation functions, the spectral density descriptions give a better specification of the random nature and of the different frequency content of the surface profile. This statement is supported, in this investigation, through selected experimental evaluations.

1.4 Scope of the Thesis

This investigation presents a unified computerized approach to describe the surface roughness. This method is based on the analysis of the surface texture as a random

process, and consequently, its best description is done by using the probability moments. Only the first and second order moments have been utilized, as this provides adequate information for both theoretical and practical considerations. The probability density function, or its related probability density function, the autocorrelation and the spectral density decomposition, are utilized.

The analog signal obtained from the output of the stylus measuring device, is converted into data points and the exact zero mean line is created. Further, the signal is normalized and standardized. The histogram, the probability distribution and the probability density functions are obtained after appropriate choice of the amplitude intervals corresponding to the data points. Then the correlation function is computed to detect the presence of periodic components, if any, and for a subsequent determination of the spectral density function. Even though the autocorrelation and the spectral density are related through a Fourier transformation and give the information in different formats, it is felt that the spectral density function is more useful for the interpretation of the results and for subsequent applications. In order to facilitate the experimental work, a direct link-up between the surface measuring system and the hybrid computer was installed. Through this link-up, digitizing of the continuous output of the measuring device was made possible and the signal could then be processed on a high-speed digital computer.

In the second Chapter, the basic components of surface irregularities are explained. A complete description with a discussion on the relative advantage of commonly employed reference systems, namely the M and the E system are also given.

The surface texture is modelled as a stochastic process in Chapter III , and the associated probabilistic descriptions are defined and explained. The relative merits of employing different stochastic functions are discussed in detail.

The experimental set-up consisting of Talysurf 4 and the commuting system for measuring and data processing of the surface texture functions are described in Chapter IV. The necessary algorithms for the evaluation of the stochastic functions are introduced and a method of digital filtering is presented.

In Chapter V the experimental results and discussions based on the computations produced in Appendices B, C, D and E are presented. A complete listing of the computer program utilized to produce the surface texture characterization as a stochastic process is given in Appendix A, together with the data points collected from the experimental work.

CHAPTER II
BASIC SURFACE MEASURING SYSTEMS

CHAPTER II

BASIC SURFACE MEASURING SYSTEMS

2.1 The Surface Geometry

The surface texture irregularities can be regarded as the superposition of three major components, namely the roughness, the waviness and the errors of form (Fig. 2.1). Most of the national standards define the roughness as the finer irregularities resulting from the inherent action of the production process, itself. The traverse feed marks and other irregularities which are in the limits of the roughness width cut-off, which is subsequently defined, are also included. The roughness width cut-off is the greatest spacing of repetitive surface irregularities to be included in the measurement of average roughness height. The second component of the surface texture is waviness. It consists of widely spaced components of the surface texture and is generally of wider spacing than roughness-width cut-off. Most frequently, waviness can occur from such factors as machine or work deflection, vibrations, chatter, heat treatment or warping strains. Roughness may be considered as being superimposed over the "wavy" surface. Finally, some other irregularities may occur at one place or at relatively infrequent or widely varying intervals on a surface; this represents the errors of form. They may include such defects as cracks, blow holes, ridges, scratches, etc. Unless otherwise specified, the effect of flaws shall not be included in the roughness height measure-

ments.

Amongst the various existing methods for assessing numerical values in the surface texture irregularities, the M and the E systems have gained more international recognition, even though the E system is accepted, from a theoretical point of view, rather than for practical use. In fact, the M and the E systems share same fundamental principle of filtering of the long waves, but the practical realization is different. The adoption of the M system in Europe and in the United States is largely influenced by the existence of appropriate instrumentation.

Some highlights of these two systems are presented in the following sections.

2.2 The M-System

The M-System [2] was initially conceived by the proposition of measuring the surface irregularity from the mean line of a sample surface texture of length L . Even though initially, the surface texture assessment in the United States was the rms-value, it has been replaced with the CLA-value. Fig. 3.1 represents the surface texture amplitude $Y(l)$ of a surface texture sample function of length L .

Let Y_i be the amplitude measured at successive discrete points. $i = 1, \dots, N$. The CLA-value, which denotes the Center Line Average, which is the basic surface texture assessment in the M-System is defined by the equation

$$[CLA] = \frac{1}{N} \sum_{i=1}^N |y_i| \quad (2.1)$$

2.3 The E-System

The E-System proposes the selection of a reference line which has a shape that follows the general trend of the surface, as opposed to the straight line in the M-System.

The reference line is better defined in an operational way, as follows:

- (i) Let a stylus with a standardized large radius scan the surface and trace the successive positions of the center of the radius as the stylus rolls over the surface.
- (ii) Shift this path up to the real profile until it touches it in such a way that the areas enclosed between this path and the real profile are minimum. This envelope line is selected as the reference line in the E-System.

The basic parameters for surface texture characterization are the Rautiefe R or the peak-to-valley maximum value, the Glattungstiefe G or R_p , or average depth, and finally the Tragenteil T_p , or bearing ratio. They are defined [4] by the following equations:

$$R_{\max} = \text{Max (peak-to-valley value)} \quad (2.2)$$

$$G = R_p = \frac{1}{N} \sum_{i=1}^N Y_i \quad (2.3)$$

$$T_a = \frac{1}{L} \cdot \Sigma [\text{of the intercepts at the level } a] \quad (2.4)$$

$$T_a = \text{the bearing ratio at the level } a \quad (2.5)$$

where the values of Y are measured with respect to the reference line previously defined.

2.4 Limitations and Critical Review of the M- and the E-Systems

The CLA value has gained some international recognition and it is in current use in Europe and in the United States. This is due to the simpler definition of the CLA- parameter and particularly, because of the development of appropriate measuring devices capable of producing on the shop floor a quick estimation of the CLA-value for different sample surfaces. However, this measure alone cannot completely specify the geometry of a given sample surface and it does not provide such information as the shape of the profile, the spacing of the crests, the bearing after a certain amount of wear, etc. The measuring device which produces the CLA- measure, automatically creates an electronic mean line of the surface. However, this mean line is not intrinsic to the surface and thus, it is not unique. Moreover, a number of different sample surfaces bearing some numerical CLA-assessments have been published in [17].

The envelope system has two major drawbacks. The first

one is associated with the inexistence of a measuring device, capable of producing the surface texture characterization based on R , G and T_a , as previously defined and the second limitation is that of high magnification factors, an impractical length of paper is needed. To portray a representative length of surface with equal horizontal and vertical magnification, in order to keep the rolling circle undistorted, the scale on which the enveloping area has to be drawn would be prohibitive.

Both systems focus on measuring only the roughness component of the surface texture. In both cases, provisions are made to produce a filtering of the long wave components of the surface texture. In the M-system, this is produced by means of an electronic filtering of the surface texture function. Current measuring devices are equipped with an electronic filtering device capable of producing three different cut-offs. In the E-System, the role of the filter is played by the radius of the rolling stylus. For the zero valued radius, the stylus will follow the profile very closely while for an infinite valued radius, the envelope will rest on the highest peak. This selection produces a filtering range from the total to zero. Standardized values for the radius of the rolling stylus and for the electronic filter, have been produced as a result of experimental analysis, rather than for sound mathematic modelling of the surface texture problem.

FIGURE 2.1
BASIC COMPONENTS OF SURFACE TEXTURE

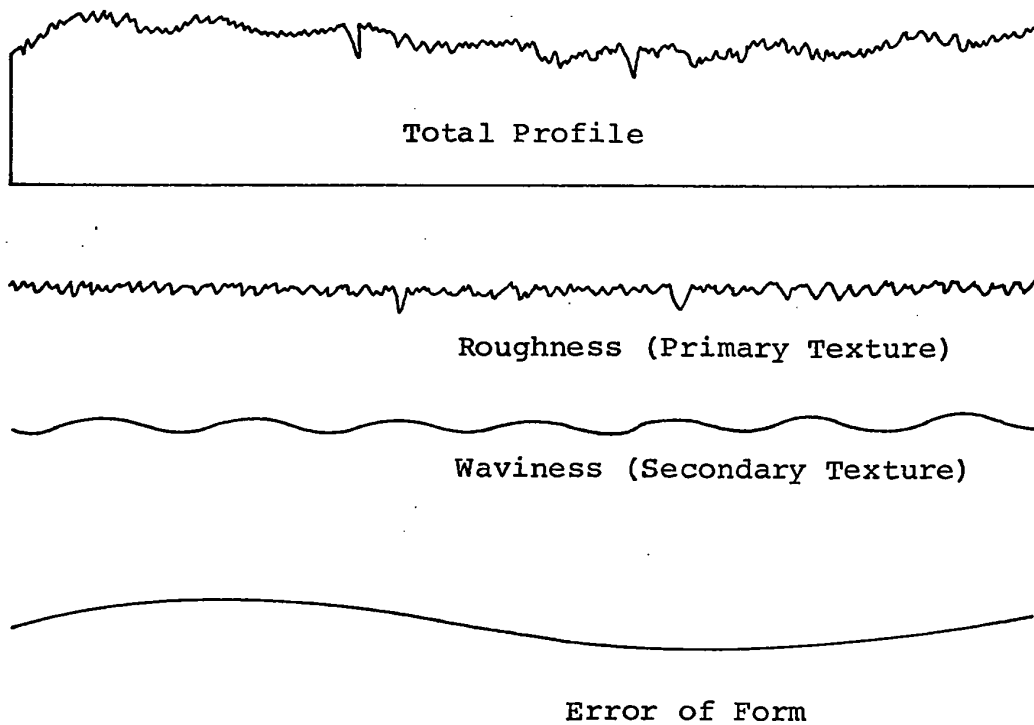


FIG. 2.1 Basic Components of Surface Texture

CHAPTER III
SURFACE TEXTURE AS A STOCHASTIC PROCESS

CHAPTER III

SURFACE TEXTURE AS A STOCHASTIC PROCESS

3.1 Introduction

This Chapter proposes a description of surface irregularities through certain basic concepts that are associated with the mathematical theory of stochastic processes. As indicated in earlier chapters, the functions required to give a satisfactory description for surfaces are: the Mean Value, the Mean Square Value, the first order Probability Functions (the distribution and the density functions), the Correlation and the Density Spectrum. These functions are introduced in the form that is relevant to surface characterization. It is argued that the Spectral Density as a single function has the best advantage of giving the most statistical information for surface profiles.

The properties of random processes such as ergodicity and stationarity are very meaningful in surface measurement and simplify greatly the experimentation part in surface data analysis. This is discussed in detail to provide information on selecting samples and sampling length. Along with the mathematical representation of the above-mentioned stochastic functions, the special case of normally distributed random processes is discussed to verify the often made suppositions that certain machining processes give surfaces having a Gaussian distribution. This information is used in Chapter IV for surface profile analysis and for

implementation in the computer program that gives the plots for the stochastic functions used for the surface texture characterization.

3.2 Surface Texture as a Stochastic Process

Any surface texture produced is the output of the machine-tool and workpiece system and consequently, it is influenced by a large number of factors, such as:

- (i) the type of machining process (milling, turning, etc)
- (ii) the dynamic characteristics of the machine-tool (stiffness, natural frequencies, damping, etc)
- (iii) the mechanical properties of the raw material (bearing resistance, elasticity, solid damping, etc)
- (iv) the environmental conditions (vibrations of the foundation, temperature fluctuation, etc)

It is not possible to produce a detailed analysis of the contribution of each of these factors in the final surface texture. Suppose ξ_i ($i=1,2, \dots$) are the contributions to the total surface irregularities from each one of the above-mentioned machining and material factors. If the factors ξ_i are taken to be mutually exclusive, the surface texture process $\{U\}$ can be regarded as the super-

position of these contributions.

$$\{U\} = \sum_i \xi_i \quad (3.1)$$

The factors ξ_i in general, for a given manufacturing process, will be functions of space and time. It is reasonable to assume that in metal cutting operations the time variable can be translated in terms of length-variables through a linear relationship. For example, this relationship can come through the feed rate and the speed. Hence, the surface process $\{U\}$ can be regarded as a simple function of space. In a two dimensional space the process takes the form $\{U\} = \{U(\ell)\}$ where ℓ is the length coordinate and U , the surface amplitude. If the number of factors ξ_i affecting $\{U\}$ is large, and if they are of equal significance, it may be concluded that $\{U\}$ will be generated as a normally distributed process, according to the Central Limit Theorem. This is true for surfaces obtained through machining methods such as grinding, lapping or through electrochemical processes.

Let $\{U\} = \{U(\ell, b)\}$ represent the surface amplitudes defined in a plane that is obtained through certain machining processes. Here, ℓ and b represent the two coordinates situated on the plane. Suppose that the removal of metal was done in successive strips of width $b = b_1$, and oriented parallel to the ℓ -axis. The amplitude description $U(\ell)_{b=b_1}$ is equivalent to a temporal record as defined in

the theory of stochastic processes. Similar temporal records can be generated for different values of the variable b . Similarly, a number of records can be generated following a pattern $U(b)_{\ell=\ell_1}$. Now the process $\{U\}$ may be taken to consist of either a collection of m paralleled-temporal records $U(\ell)$ or as n -ensemble directional records $U(\ell)$. That is

$$\{U\} = \begin{array}{c} \begin{array}{|c|} \hline U_1(b) \\ \hline \end{array} \\ \begin{array}{c} j=1,2,\dots,n \\ \hline \begin{array}{|c|} \hline U_1(\ell) \quad U_{11} \quad \dots \quad U_{1j} \quad \dots \quad U_{1n} \\ \hline \vdots \\ U_{j1} \quad \dots \quad U_{ij} \quad \dots \quad U_{in} \\ \vdots \\ U_{m1} \quad \dots \quad U_{mj} \quad \dots \quad U_{mn} \\ \hline \end{array} \\ \hline i=1,2,\dots,m \end{array} \end{array} \quad (3.2)$$

The stochastic process $\{U\}$ can then be easily described through $\{U_i(\ell)\}$ or as $\{U_j(b)\}$ depending on the nature of the measurement of the surface texture process. On the basis of this argument it may now be proposed that the process under consideration is given by

$$\{U\} = \{U_i(\ell)\} \quad (3.3)$$

For a comprehensive description of the surface texture process, it is necessary to consider all the records $U_i(\ell)$, as i varies from zero to infinity. Due to human limitations, certain reasonable maximum values of i may have to be chosen. Thus, the entire ensemble of records will be

characterized by a finite number of temporal records $U_i(\ell)$ where $i=1,2,\dots,n$ (i being finite). The stochastic description of the surface process is obtained by computing the probability functions and the associated probability moments. For instance, the ensemble mean value, mean square value and auto-covariance are given by the following equations:

$$\langle U(\ell_1) \rangle = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n U_i(\ell_1) \quad (3.4)$$

$$\rho_u(\ell_1, \ell_1) = \langle [U_i(\ell_1) - \langle U(\ell_1) \rangle]^2 \rangle \quad (3.5)$$

$$\rho_u(\ell_1, \ell_2) = \langle [U_i(\ell_1) - \langle U(\ell_1) \rangle][U_i(\ell_2) - \langle U(\ell_2) \rangle] \rangle \quad (3.6)$$

where the angular brackets $\langle \dots \rangle$ denote averaging over the ensemble.

Experimental results have revealed that these parameters, computed for the surface texture characterization, possess almost the same numerical values, independent of the selection of the origin of the variable ℓ . In other words, any ensemble record taken at a location ℓ_1 , selected arbitrary over the sample length, will produce the same statistical identifications. This is the property of stationarity of a random process and through this concept, all the ensemble averages become independent of the origin of the ℓ -axis and is true for the entire process $\{U_i(\ell)\}$.

The surface texture process is said to be weakly station-

ary if only the ensemble mean value, mean square value and auto-covariance do not depend upon the arbitrary selection of the origin of ℓ . In other words:

$$\langle U(\ell_1) \rangle = \langle U(\ell_1 + \lambda) \rangle \quad (3.7)$$

$$\rho_U(\ell_1, \ell_1) = \rho(\ell_1 + \lambda, \ell_1 + \lambda) \quad (3.8)$$

$$\rho_U(\ell_1, \ell_2) = \rho(\lambda) \text{ where } \lambda = \ell_1 - \ell_2 \quad (3.9)$$

A more restrictive concept is that of a strong stationarity which requires that all possible statistical moments are not affected by a translation of the length origin. However, this condition is prohibitive for many engineering applications and is accommodated only by Gaussian distributed records.

The surface texture characterization by means of ensemble statistical quantities, require the use of a large number of sample functions say $n \rightarrow \infty$, and this may not always be possible or economically justified. It is possible, however, to select one representative sample function $U_1(\ell)$ with the understanding that the probability of choosing a non-representative sample is very small, and to compute the corresponding statistical description by averaging over the variable ℓ . These quantities are generally known as the "temporal" averages, by analogy to those processes where the independent variable is time instead of length. For example, the temporal mean value and auto-covariance are given

by:

$$\bar{U}_i = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^n U_{ij} \quad (3.10)$$

$$R_{U_i}(r) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^{n-r} U_{ij} U_{i,(j+r)} - (\bar{U}_i)^2 \quad (3.11)$$

where $R(r)$ is the temporal auto-covariance computed for a lag interval r .

If the statistical ensemble averages are equal to the statistical temporal averages, the stationary process U is said to be ergodic. This condition may result from equating equations (3.4) to (3.10) and (3.6) to (3.11), i.e.:

$$\langle U(\ell_1) \rangle = \bar{U}_i \quad (3.12)$$

$$\rho_U(\ell_1, \ell_2) = R_U(\lambda), \quad \lambda = \ell_1 - \ell_2 \quad (3.13)$$

As in the case of stationarity weak ergodicity requires that the process $\{U\}$ should be stationary and that the ensemble and the temporal mean values and auto-covariances should be equal. Strong ergodicity requires that all the temporal and ensemble statistics be equal. For Gaussian distributed processes such as ground, lapped or electrochemically produced surfaces, weak ergodicity implies automatically a strong ergodicity. The importance of the property of ergodicity in relation to the surface texture characterization relies in that the entire process $\{U\}$ representing the

amplitudes of the irregularities can effectively be described by simply producing the stochastic analysis of only a few temporal records.

3.3 The Probability Description of the Surface Texture

The surface texture which in most of the cases may be taken to be an ergodic random process (which automatically includes stationarity), can be characterized completely by the probabilistic functions computed for a sample record as represented in Fig. 3.1, where the amplitudes U are plotted as a function of the independent variable x . A complete description is only possible by generating an infinite number of probability moments. However, for both analytical and practical considerations, the following parameters and functions may provide satisfactory results: the mean value, the mean square value, the first order probability functions (the density p and the distribution P), the auto-correlation R (or the auto-covariance ρ), and finally the spectral density S .

3.3.1 Mean and Mean Square Value

The mean and the mean square values are both basically arithmetic averages, usually well defined numerical values. They represent the magnitude of the steady state component of the surface texture, over which is superimposed a fluctuating component of a general intensity equal to the deviation σ .

Let U be the function which is collected as the output data of the measuring device, and which is proportional to the surface amplitude irregularities. Further, let this function be collected over a large observation (or sample) length L . Then the temporal mean value is given by:

$$\bar{U} = \lim_{L \rightarrow \infty} \frac{1}{L} \int_0^L U(\ell) d\ell \quad (3.14)$$

and the temporal mean square value is:

$$\bar{U}^2 = \lim_{L \rightarrow \infty} \frac{1}{L} \int_0^L U^2(\ell) d\ell \quad (3.15)$$

Through a linear transformation, the ℓ -axis can be shifted by the quantity \bar{U} to obtain a process $Y(\ell)$ which has a zero mean value.

$$Y(\ell) = U(\ell) - \bar{U} \quad (3.16)$$

and

$$\bar{Y} = 0 \quad (3.17)$$

The mean square value of Y , denoted by \bar{Y}^2 is:

$$\bar{Y}^2 = \sigma_Y^2 = \lim_{L \rightarrow \infty} \frac{1}{L} \int_0^L [U(\ell) - \bar{U}]^2 d\ell \quad (3.18)$$

These equations are used in Chapter IV for processing of data for surface texture characterization as a stochastic process.

The mean value is useful for the definition of a reference system which is dependent upon the sample function. This is done by using the equation (3.16) and the operation is known as the normalization of the record U into the normalized record Y . This operation further produces some mathematical simplifications. For example the equation of the autocorrelation function

$$R_U(\lambda) = \rho_U(\lambda) - (\bar{U})^2 \quad (3.19)$$

becomes

$$R_Y(\lambda) = \rho_Y(\lambda) \quad (3.20)$$

since \bar{Y} which should appear in the right-hand side of the equation is equal to zero.

3.3.2 The Variance σ^2

The variance is defined by the equation (3.18) and it is the mean square value of the amplitudes measured above the mean line of the record. It may be used for a rough characterization of the surface texture and secondly, it is also useful in obtaining a relative comparison of surface amplitudes. For example, consider two records Y_1 and Y_2 having corresponding variances σ_1^2 and σ_2^2 . Let σ_1 be much smaller than σ_2 . Then according to equation (3.18)

$$\frac{1}{L} \int_0^L Y_1^2(\ell) d\ell \ll \frac{1}{L} \int_0^L Y_2^2(\ell) d\ell$$

and consequently $Y_1(\ell)$ is much smaller than $Y_2(\ell)$ for most of the cases.

The second application of the variance is related to the process of data preparation through the standardization of the record Y that has been already normalized. The new record Z is obtained by the equation

$$Z(\ell) = \frac{Y(\ell)}{\sigma_Y} \quad (3.21)$$

it is characterized by the fact that its mean value is zero and its variance equals one. The record Z is called the normalized and standardized version of the original record U , which may have been collected at the output of a surface texture measuring device. The use of Z rather than U or Y introduces mathematical simplifications such as the standardization of the values of the autocorrelation at a zero lag value and the standardization of the areas enclosed between the spectral density plot and the frequency axis.

3.3.3 The Amplitude Density Function

By dividing the amplitude scale Z into small parts ΔZ , the density of the amplitudes found within the interval ΔZ can be obtained by measuring for how long the function Z has amplitude values between the levels Z and $Z + \Delta Z$ relative to the total length L over which the surface is being investigated.

$$p_z(Z, \Delta Z) = P[Z \leq Z(l) \leq (Z + \Delta Z)] \quad (3.22)$$

$$p_z(Z) = \lim_{\Delta Y \rightarrow 0} \lim_{L \rightarrow \infty} \frac{1}{L} \left[\sum_{i=1}^L \ell_i \right] \quad (3.23)$$

By varying the value of Z from $-\infty$ to $+\infty$, the amplitude density $p_z(Z)$ can be plotted as a function of Z . In the case of a normally distributed random function Z , with a zero mean value and known variance σ_y^2 , the amplitude density is given by the Gaussian distribution curve. The normalized amplitude density plot obeys the equation

$$p_z(Z) = \frac{1}{\sigma_z \sqrt{2\pi}} \text{EXP} [-Z^2/(2\sigma^2)] \quad (3.24)$$

The amplitude distribution function $P(Z)$ is related to the distribution function $p(Z)$ by

$$P_z(Z) = \int_{-\infty}^Z p(\xi) d\xi \quad (3.25)$$

The density function $p(Y)$, if known, is very useful in generating all the probability moments which are given by

$$M_z^{(k)} = \int_{-\infty}^{+\infty} Z^k p(\xi) d\xi \quad (3.26)$$

The mean value is the first order moment and therefore

$$\bar{Z} = m_z^{(1)} = 0 \text{ by definition of } Z \quad (3.27)$$

The Center Line Average is a parameter utilized for the characterization of the surface texture irregularities and it

represents the arithmetic average of the absolute values of the amplitudes Z . It may be defined as the first order moment of the function $|Z|$. Then

$$[CLA]_Z = \int_{-\infty}^{+\infty} |Z| p(\xi) d\xi \quad (3.28)$$

while (3.27) is zero by definition, equation (3.28) bears definite numerical values. It represents the height of a rectangle of length L (the length of the sample function), and an area equal to that enclosed between the surface function and the zero mean line axis.

The second moment is the variance σ^2 already defined by the equation (3.18). For the function Z

$$\sigma_Z^2 = m_Z^{(2)} = \int_{-\infty}^{+\infty} Z^2 p(\xi) d\xi = 1 \quad (3.29)$$

In the practice of computing the probability density one has to consider an amplitude domain of interest say $(A - B)$, the number of class intervals, k , into which this domain is to be subdivided to give the so-called amplitude class intervals and the length, L , of the record to be investigated.

By varying Z from A to B and by plotting $p(Z)$ the surface texture amplitude characterization is produced. Four such plots which correspond to four length-histories (Fig. 3.2) are represented in Fig. (3.3). Surface texture obtained by such processes as grinding, lapping or electrochemical machining follow a Gaussian distribution.

The equations (3.23), (3.25), (3.27) and (3.29) are

utilized in the experimental analysis and they are incorporated in the computer program given in the Appendix.

The description of the surface texture by using the amplitude density distribution functions has a limitation in that it cannot provide information regarding the behaviour of Z in the lengthwise axis direction. The second limitation is that the probability functions are not uniquely defined and their shape depends upon the values of the parameters A , B and k . For this reason, the amplitude description of the surface texture must be supplemented by introducing the autocorrelation and the spectral density functions.

3.3.4 The Autocorrelation Function

The autocorrelation function of $Z(\ell)$ characterizes the general dependence of the standardized amplitudes $Z(\ell)$ on the values of $Z(\ell + \lambda)$. An estimate of the autocorrelation can be obtained by taking the product $Z(\ell) Z(\ell + \lambda)$ and averaging this, over the observation length L . As the value of L tends to infinity, the resulting average tends to the true autocorrelation function.

$$R_Z(\lambda) = \lim_{L \rightarrow \infty} \frac{1}{L} \int_0^L Z(\ell) Z(\ell + \lambda) d\ell \quad (3.30)$$

The autocorrelation $R(\lambda)$ is always a real, even function with a maximum value at the origin.

$$R(\lambda) = R(-\lambda) \quad (3.31)$$

$$R(0) \geq R(\lambda)$$

The mean and the mean square values are related to the autocorrelation function

$$\bar{Z} = \sqrt{R_Z(\infty)} \quad (3.32)$$

$$\sigma_Z^2 = \bar{Z}^2 = R_Z(0)$$

The autocorrelation function discriminates between purely random and periodic components masked on a random background and it can provide a general indication regarding the degree of randomness of the original function $Z(\ell)$. An important feature of the autocorrelation relies in that the autocorrelation of a sine wave is a cosine type function which persists periodically over all the length displacement λ , and has the same period as the underlying sine wave. However, the phase information is lost. The autocorrelation of a sine wave plus random noise is simply the sum of the autocorrelograms for the sine wave and for the random noise separately. The shape of the autocorrelation depends on the frequency composition of the surface texture. For a wide band noise function the autocorrelation diminishes rapidly to zero while for a narrow band case, it is a decaying version of the sine wave autocorrelation.

The autocorrelation can give a rapid check of the

stationarity of the function Z (and by that of the original function U), by observing the variation of R as the length of the record diminished from L to L_1 . By performing different shifts in the origin of the record and comparing the autocorrelograms, it can be directly observed whether the sample process $Z(\ell)$ possesses stationarity characteristics or not. This will also give the error in such an assumption. A plot of R can also reveal the presence of harmonic or of deterministic components in the surface texture function. A third application of the autocorrelation is for the computation of the spectral density function which is related through a Fourier transform. The typical autocorrelogram plots which correspond to sample functions given in Fig.(3.2) are given in Fig.(3.4). They represent the autocorrelograms of: (a) a sine wave, (b) a sine wave plus random noise, (c) the narrow band random noise, and finally, (d) the wide band random noise. Their corresponding Fourier transforms are indicated in Fig. (3.5).

3.3.5 The Spectral Density

The spectral density of the surface texture is defined as the Fourier transform of the autocorrelation function R

$$S(f) = 2 \int_0^{\infty} e^{-j2\pi f\lambda} R(\lambda) d\lambda \quad (3.33)$$

$$R(\lambda) = \frac{1}{2} \int_0^{\infty} e^{+j2\pi f\lambda} S(f) df \quad (3.34)$$

where f is the frequency in cps. For a real process such as a surface texture, the equation (3.33) can be rewritten as

$$S(f) = 4 \int_0^{\infty} \cos(2\pi f\lambda) R(\lambda) d\lambda \quad (3.35)$$

This formula is used in the experimental analysis and it is incorporated in the computer program described in Chapter 4.

The mean and the mean square values are also related to the spectral density

$$\bar{Z}^2 = \sigma_Z^2 = \lim_{L \rightarrow \infty} \frac{1}{L} \int_0^L S_Z(f) df \quad (3.36)$$

Assume that a signal $Z(\ell)$ is passed through a band pass filter of narrow band width so that only a portion of the function Z which has that frequency band width will be traversing it. The square of the output represents the mean square value of the filtered signal for a relatively small bandwidth. Using a sweeping filter, capable of covering the entire frequency domain from zero to $+\infty$, in very small increments, and plotting the successive squared values of the observed output, then the spectral density of Z is obtained. This simply means that the spectral density function produces the general frequency description of the surface texture Z in terms of the mean square value of the various harmonic components. It is obvious that the wave length associated with a frequency f is equal to $1/f$. Then the frequency decomposition of the surface texture can easily

be regarded as the corresponding wave length decomposition.

That is

$$S_Z(f, \Delta f) = \lim_{L \rightarrow \infty} \frac{1}{L} \int_0^L Z^2(\ell, f, \Delta f) d\ell \quad (3.37)$$

where $Z(\ell, f, \Delta f)$ represents that portion of the surface texture function Z , of frequencies enclosed in the interval f and $f + \Delta f$. For small values of Δf the following equations hold

$$S_Z(f, \Delta f) = S_Z(f) \Delta f \quad (3.38)$$

$$S_Z(f) = \lim_{\Delta f \rightarrow 0} \lim_{L \rightarrow \infty} \frac{1}{L} \int_0^L Z^2(\ell, f, \Delta f) d\ell \quad (3.39)$$

It should be remarked that the spectral density of a sine-wave is a delta function at the frequency f_0 of the sine and zero elsewhere. Even so, the integral of the function $S(f)$ over any frequency range that includes the nominal frequency f_0 , has a finite value, in general equal to the variance. For a wide band function, the spectral density is smooth and broad. By reducing the size of the frequency domain of the band, the spectral density becomes sharply peaked and at a limiting case, it becomes a delta function.

Suppose a signal is such that it is a superposition of periodic components such as waves, squares or triangular waves and a random component. In this case, the spectral density is the summation of the corresponding spectral plots

of these components. The final plot is in general characterized by a peak in the region of the frequency of the periodic component. Some typical spectral density plots, corresponding to the length histories given in Fig. (3.2) are presented in Fig. (3.5).

3.4 Conclusions

From the presentations in this Chapter, it may be seen that besides the basic stochastic descriptions such as the mean and the mean square value, the spectral density functions provide an effective method of characterizing the surface texture in terms of a frequency decomposition. On the basis of the spectral density it is possible to arrive at some relation between the machining operation and the surfaces produced. For example, grinding, lapping and electrochemical finishing operations, induce response in the tool that has in general, a flatter spectral density curve, compared to the turning and the milling operations (especially for heavy cuts) which exhibit sharply peaked spectral densities. These are directly reflected on the surfaces produced and hence, are useful in establishing the parameters for the characterization. Furthermore, for surfaces that have random components that are superimposed on deterministic periodic components, spectral density gives a straightforward description. Hence, the spectral density can be used to detect the presence of predominant deterministic components and is widely used on computations that follow in Chapter IV.

FIGURE 3.1
SURFACE TEXTURE SAMPLE RECORD

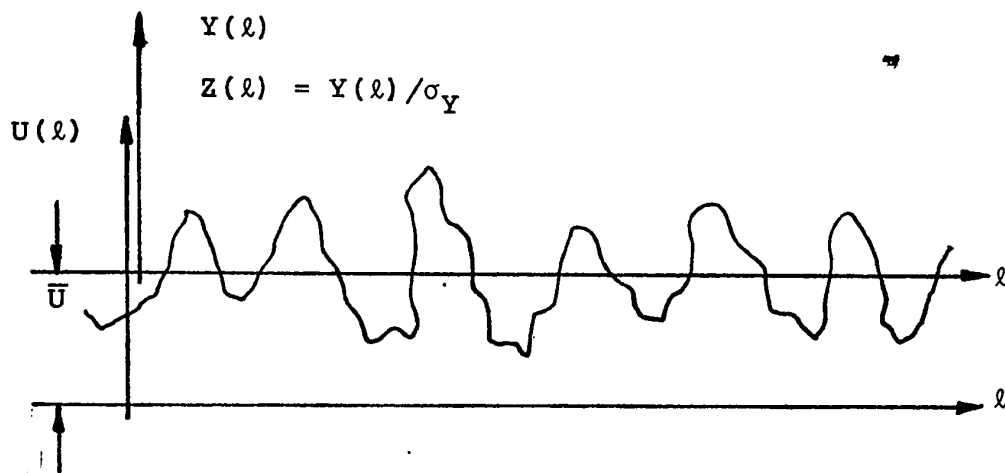


FIG. 3.1 Surface Texture Sample Record

- $U(l)$ is the output collected from the measuring device
- $Y(l)$ is the normalized version of $U(l)$
- $Z(l) = Y(l) / \sigma_Y$ is the normalized and standardized version of $U(l)$

FIGURE 3.2
FOUR SPECIAL LENGTH HISTORIES
Sample Plots

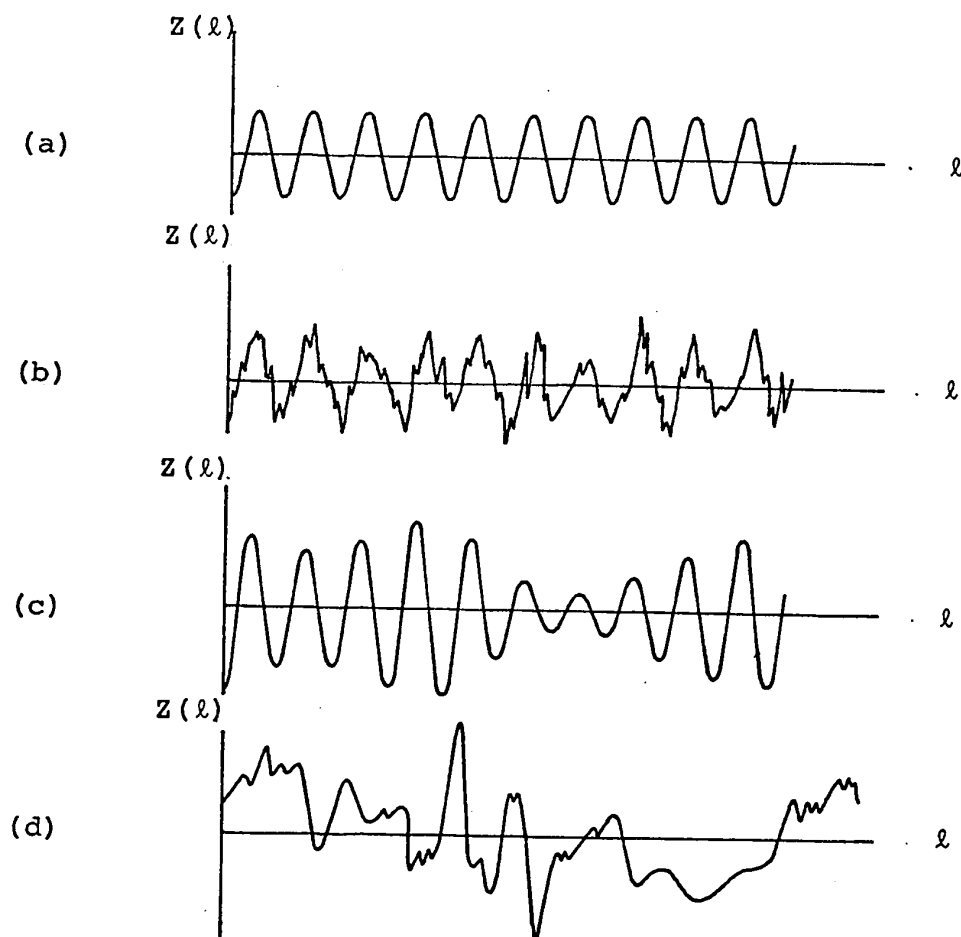


FIG. 3.2 Four Special Length Histories, [29]

Sample Plots

- (a) Sine wave
- (b) Sine wave plus random component
- (c) Narrow band random component
- (d) Wide band random component

FIGURE 3.3
FOUR SPECIAL LENGTH HISTORIES (FIG. 3.2)
Probability Density Function Plots

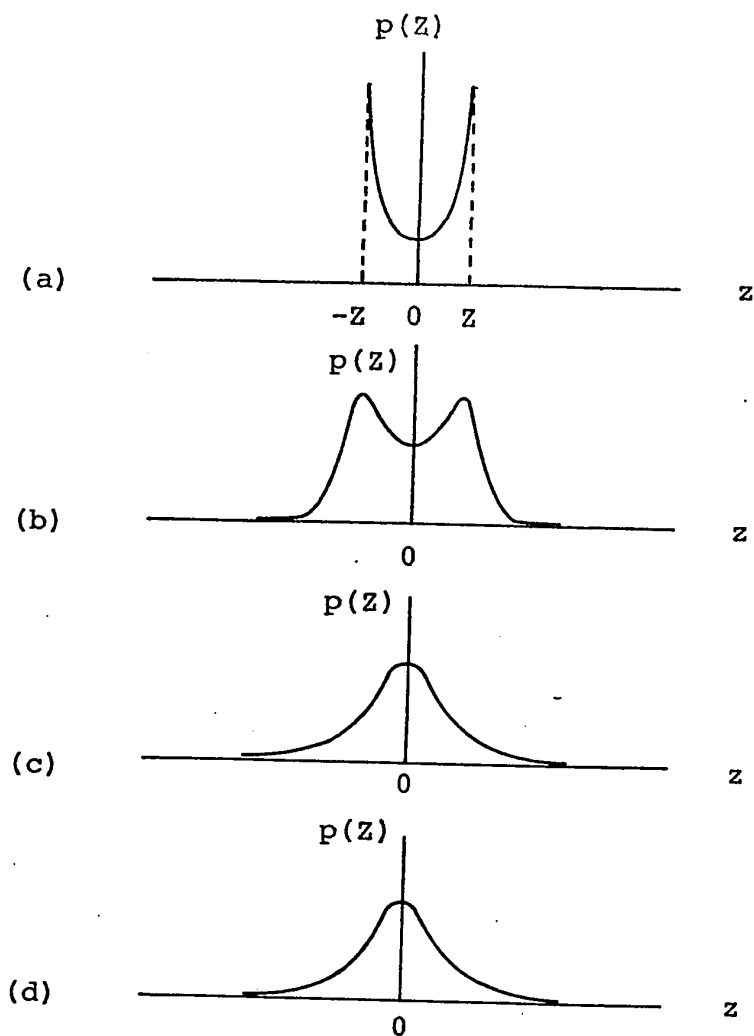


FIG. 3.3 Four Special Length Histories (Fig. 3.2), [29]
Probability Density Function Plots

- (a) Sine wave
- (b) Sine wave plus random component
- (c) Narrow-band random component
- (d) Wide-band random component

FIGURE 3.4

FOUR SPECIAL LENGTH HISTORIES (FIG. 3.2)

Autocorrelation (Autocorrelogram) Function
Plots

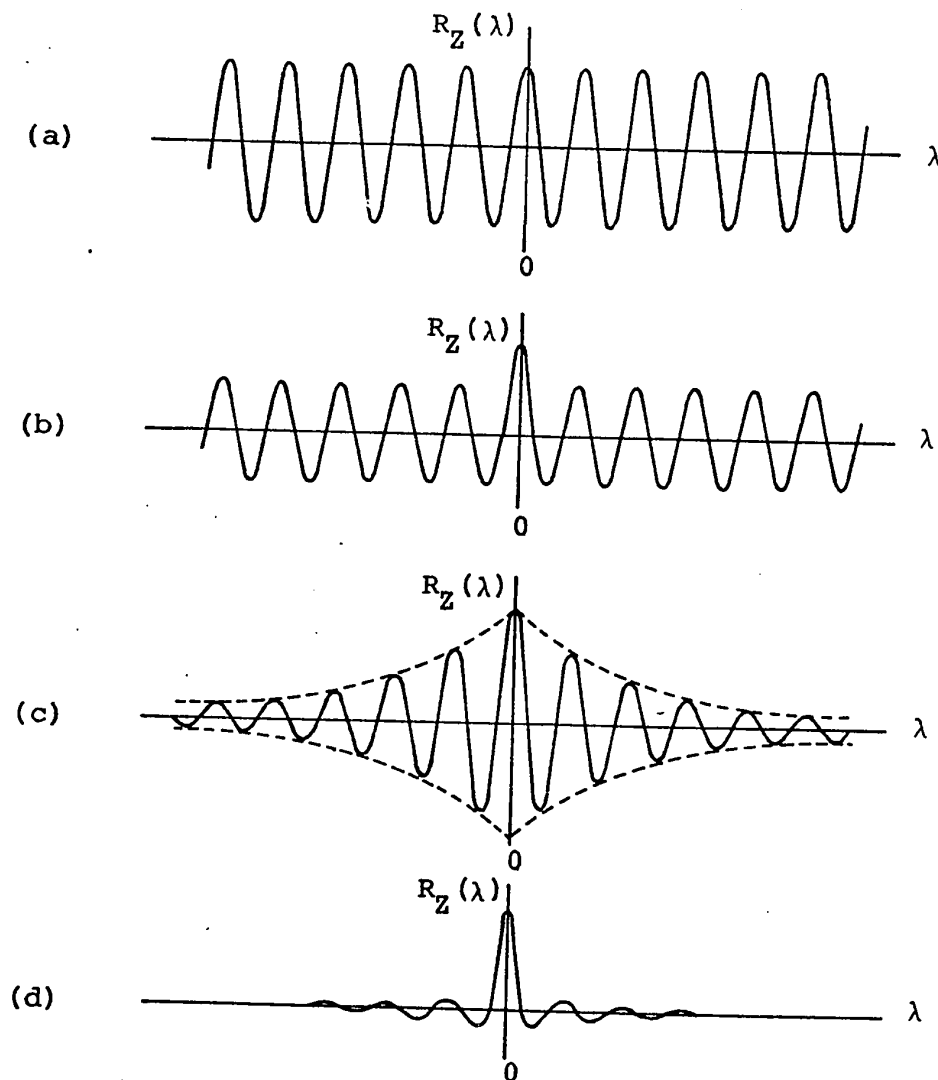


FIG. 3.4 Four Special Length Histories (Fig. 3.2), [29]

Autocorrelation (Autocorrelogram) Function Plots

- (a) Sine wave
- (b) Sine wave plus random component
- (c) Narrow-band random component
- (d) Wide-band random component

FIGURE 3.5
FOUR SPECIAL LENGTH HISTORIES (FIG. 3.2)
Spectral Density Function Plots

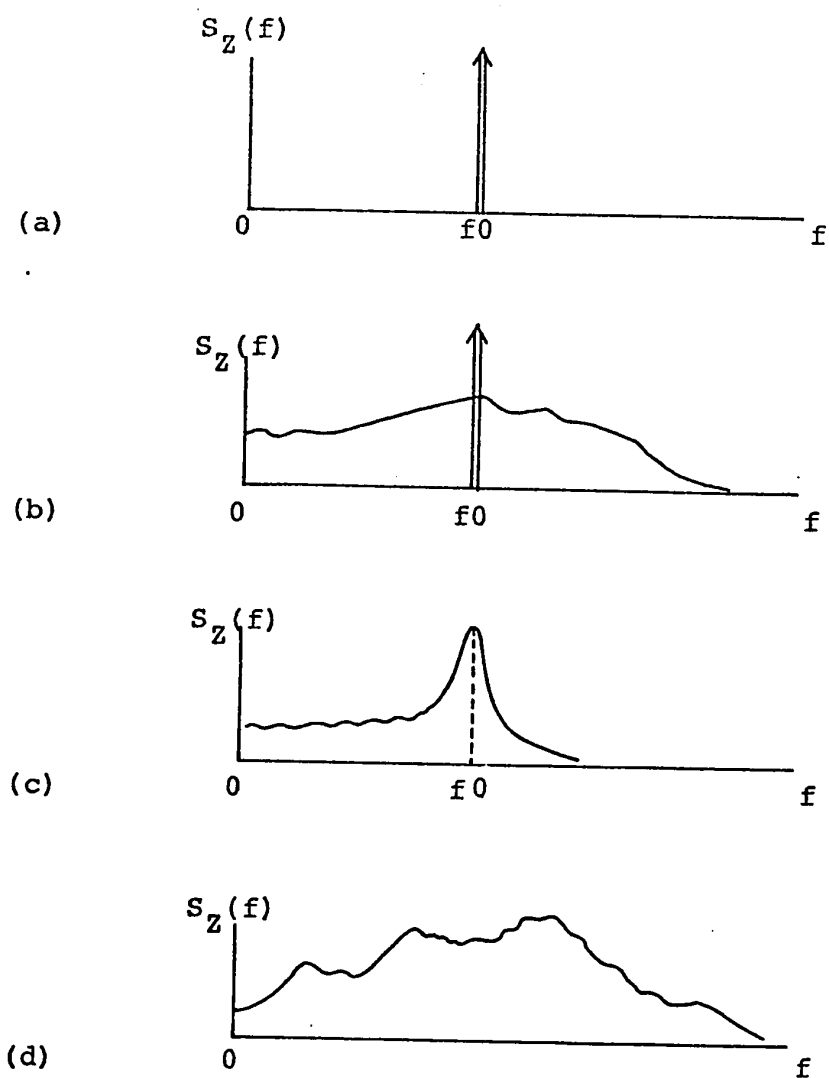


FIG. 3.5 Four Special Length Histories (Fig. 3.2) [29]

Spectral Density Function Plots

- (a) Sine wave
- (b) Sine wave plus random component
- (c) Narrow-band random component
- (d) Wide band random component

CHAPTER IV

THE EXPERIMENTAL WORK AND DATA ANALYSIS

CHAPTER IV

THE EXPERIMENTAL WORK AND DATA ANALYSIS

4.1 Introduction

To compute the different stochastic functions defined in Chapter III, certain algorithms are necessary for employment in specifying manufacturing processes. A computer program is developed to provide automatically the key descriptors for such surface texture specifications. The data for the computation is obtained through an experimental set-up employing a surface measuring equipment, which is linked to a hybrid computer for the processing of the output signal.

The Talysurf 4 is a surface texture analyzer. A stylus scans the surface texture, while a transducer converts the vertical movement into an electrical signal U . This is then processed in an electronic unit and is given as the output signal. The Talysurf has the facility of:

- (i) plotting the magnified surface texture amplitudes on a Y-t strip-chart recorder;
- (ii) computing the CLA-value and finally,
- (iii) to provide an electrical output for further data processing.

The electrical output of the Talysurf 4 is recorded on an analog magnetic tape recorder capable of input voltages smaller than $\pm 3V$. The frequency resolution of this recorder can

be adjusted by simply changing a plug in the printed circuit board. A resolution of 1250 cps was utilized in the experiments.

The hybrid computer system EAI-690 is programmed and provides the sampling and the data acquisition of the analog signal collected from the Talysurf or from a magnetic tape storage. Its main peripheral equipment consists of a high speed paper tape puncher and reader. Because of the limited memory (8k) of the hybrid computer system as compared to the size of the computer program and the amount of data points to be analyzed, the CDC-3300 computer was utilized. A telephone communication line between these two systems was utilized to convey the data points of the surface texture function into an appropriate file on the CDC-3300 computer. These data points are processed with a specialized program capable of making the computation of the statistical functions discussed in the previous Chapter.

The computer program used for producing the stochastic descriptions is detailed in this Chapter, along with the algorithms employed. The listing of the program and the experimental data are given in the Appendices.

4.2 The Principle of the Talysurf 4

The Talysurf 4 ^[31] is a surface texture analyzer which consists of a motor driven inductive pick-up which is connected to an electronic unit and to a strip chart or a Y-t record-

er. A sharply pointed stylus traces the profile of the surface irregularities. The pick-up is an inductive type transducer in which the vertical movement of the stylus modulates a carrier wave form. The signal is amplified, demodulated and then used to operate a strip chart recorder. The signal is also passed through a built-in filtering device and through an integrator capable of producing the CLA-value. The block diagram of the Talysurf 4 is presented in Fig. 4.1 .

4.3 The Meter Cut-Off of Talysurf 4

Before introducing the meter cut-off of the Talysurf 4, the following explanations on the roughness and waviness may be relevant.

Roughness is defined to represent the irregularities of the surface texture that result from the inherent action of the production process itself. These include traverse feed marks and the irregularity within them.

Waviness is defined as that component of the surface texture upon which the roughness is superimposed. Waviness results from such factors as machine or work piece deflection, self-induced vibrations such as chatter, heat treatment or warping strains.

The output signal collected from the Talysurf 4, which consists of a superposition of roughness and waviness, is amplified and demodulated and then is directed to the Y-t

recorder or to the built-in filtering device, as shown in Fig. (4.1). The filters are intended to eliminate the waviness and to let through only the roughness part of the surface which is then integrated to produce the CLA-value.

To obtain a proper measure of a surface, it is necessary to select a sampling length L that is long enough to include the various roughness crest spacings exhibited by the surface. In the electrical integrator, the meter cut-off is generally made equal to the sampling length chosen.

In the particular case of Talysurf 4, there is a set of three cut-offs corresponding respectively to the wave-length of 0.1 in (2.5 mm), 0.03 in (0.8 mm) and 0.01 in (0.25 mm). In order to have a proper relationship between the meter cut-off value and the length of surface sample function, the "Operator Instruction Book" [31], recommends that the CLA value, for a cut-off of 0.1 in (2.5 mm), should be computed by using a sample length of 10 mm. Similarly, the cut-off values of 0.03 in (0.8 mm) and 0.01 in (0.25 mm) are associated with sample lengths of 5 mm and 2.5 mm respectively. Defining the sampling-length as the minimum length which is necessary in order to produce an adequate characterization of a surface, it is found experimentally that for values of 10, 5 and 2.5 mm, the sampling length is contained from 3 to 10 times in these figures.

The specifications of the filters in the electronic unit of Talysurf 4 is conformed to the requirements laid down by the British, U.S. and Canadian standards, so that they do not

give a sharp cut-off in the measurement of the irregularities of sampling length greater than the specified meter cut-off. The characteristic curves of the filters, utilized in the Talysurf 4, are shown in Fig. (4.3). As an illustration of the use of these charts, consider a surface having an irregularity spacing of 0.03 in or 0.8 mm. With a meter cut-off of 0.1 in or 2.5 mm, the chart shows that almost the whole amplitude of the irregularity will contribute to the CLA - meter indication. With a cut-off 0.03 in or 0.8 mm, these irregularities will contribute only 75 percent of their amplitudes to the meter reading.

4.4 The Experimental Set-Up

The surface texture is technically considered to be the superposition of three components: the roughness, the waviness and the errors of form. The Talysurf 4 measuring device senses the sample surface and produces one of the following types of outputs:

- i) a recording of the surface texture graph
 - ii) the value of the CLA-value
 - iii) an auxiliary electronic signal
-
- i) The recording is a magnified version (up to $\times 100,000$) of the surface texture irregularities. It represents a true and an unmodified version of the surface texture irregularities since the electrical signal driving the strip-chart recorder

is collected in the electronic unit before entering the filtering device. The values of the magnification factors of the Talysurf 4 are indicated in Fig. (4.4).

ii) The computed CLA-value can be obtained on the CLA-meter if the stylus is set to travel at the appropriate speed indicated by the letters "AV" which means - averaging. The measurements can be done in three filtering set-ups. There are three cut-offs corresponding to a meter cut-off of 2.5, 0.8 and 0.25 mm, respectively. The CLA-value so obtained, represents the official numerical assessment of the surface texture in British, U.S. and ISO - standards.

iii) The auxiliary electronic signal represents the third possible output of the Talysurf. This electrical signal is collected at two particular points in the electronic unit, namely before and after the built-in filtering device providing thus the possibility of using the original meter cut-off. For the experimental work, only one electrical connection was available, namely the one before the filters. The electrical output, in general, is necessary for the purpose of automatic data processing in view of the computation of the stochastic descriptors. More details on this are given in the next section.

4.4.1 Description of the Measuring System

The surface measuring device - Talysurf 4, generates an electrical signal which is proportional to the stylus move-

ment on the surface texture irregularities. This signal is recorded on a magnetic tape (box 2 in Fig. (4.5) and collected in the tape-bank on the surface texture information.

The tape recorder feeds the surface function into the hybrid-computer system EAI-690 (box 3 *ibid*), which is used as a data acquisition device and produces a coded sample version of the analog signal (recorded on the magnetic tape). This discrete version is punched on paper tape and it is ready for further data processing and computations.

An experimental telephone data communication system (boxes 4 and 5 *ibid*) was established between two computers, the EAI-690 (box 3) and the CDC-3300 (box 6) for the transmission of the sampled data into an appropriate file of the high speed computer.

After storing the transmitted data in the CDC-3300, the program which is described in the following section is called in to produce a complete statistical analysis of these data points, as discussed in Chapter III. The following arithmetic quantities and functions are computed:

- i) the mean value
- ii) the mean square value
- iii) the probability density and distribution, and
- iv) the autocorrelation function and the spectral density

4.5 Computer Program for Stochastic Modelling

As shown in Fig. (4.5) the surface texture function is sampled in unit 3 and then transmitted to unit 6 via a telephone communication line. A punched version of the transmitted data is used for experimental purposes.

The computer program that computes the stochastic parameters, accepts the input as punched cards in the Binary Coded Decimal version. The input data represents the digitized version of the surface texture function and it is expressed in the machine units, where one machine unit is equal to 10 volts. It is then necessary to convert the voltages into equivalent millimeters and this operation is implemented in the program. The operation of normalization and standardization are performed next. These reduce the function U to a function Z having a zero mean value and a standard deviation which is equal to unity.

The process of computation takes place in the program in four major steps:

- i) the data preparation, including the data-input, the data scaling, the calculation of the mean and of the mean square value and the operation of normalization and standardization;
- ii) the second step is the computation of the probability functions (or the amplitude statistics) such as the probability density and the probability

- distribution;
- iii) the computation of the stochastic functions, the autocorrelation and the spectral density and finally;
 - iv) the output of the results in terms of plots and tables.

A number of parameters have to be defined prior to the computation of the stochastic descriptors of the surface texture. Some of these parameters are the following: the number of data points N , the amplitude interval $[A,B]$, the maximum autocorrelation lag number, the sampling time, etc.

The flow charts and the detailed computer listing are included in Appendix A, together with the experimental results (Appendix B,C,D,E). The algorithms utilized in writing this program are detailed in the following section.

4.6 Algorithms for Digital Computation

After collecting the output from the measuring service, the Talysurf 4, in analog form, it is necessary to produce its sampled version and to submit the digitized form for further statistical computations. The principal algorithms utilized in the computer program for computation of the functions introduced in Chapter III are presented in this section.

4.6.1 Sampling of the Analog Signal

This operation consists of converting the continuous output taken from the Talysurf 4, into a sequence of discrete amplitudes. This can be achieved by setting up the system to produce n -samples per second or conversely, to produce a reading of the analog signal every $1/n$ seconds. The maximum observable frequency in the original signal is then:

$$f_c = \frac{1}{2h} \quad \text{where} \quad h = \frac{1}{n} \quad (4.1)$$

The frequency f_c is known as the Nyquist or the cut-off frequency. The frequencies in the input data which are higher than $\frac{1}{2h}$ will then be folded and diffused on a lower frequency range from zero to $\frac{1}{2h}$. However, if h is sufficiently small (or f_c is relatively large) it may be expected that no frequencies in the data exist beyond the f_c frequency.

The quantization of the analog signal into discrete values introduces relatively small errors and according to Bendat [29], if the amplitude scale consists of more than 256 divisions, then the standard deviation of the quantization error to signal ratio is equal to $\frac{0.29}{256} = 0.001$, and thus it is negligible for most cases. The use of a digital voltmeter that is capable of providing at least three significant digits was found satisfactory for the present experiment.

4.6.2 The Calculation of \bar{U}

This parameter already defined by the equation (3.14) is computed over the sampled function U_n where

$$U_n = U(\ell_0 + \lambda n)$$

where ℓ_0 is the starting point, λ is the distance between successive points and n is an integer varying from zero to N .

$$\bar{U} = \frac{1}{N} \sum_{n=1}^N U_n \quad (4.2)$$

The significance of the mean value has already been presented in Chapter III.

4.6.3 The Normalization of Zero Mean Value

This operation consists of converting the function U into a function Y (see also Equation (3.16)) of zero mean value. This is done by subtracting \bar{U} from all the values of U_n

$$Y_n = U_n - \bar{U} \quad (4.3)$$

and hence $\bar{Y} = 0$.

4.6.4 Calculation of Mean Square Values

This parameter defined by Equation (3.18), can be computed for the normalized function Y

$$\bar{Y}^2 = \sigma_Y^2 = \frac{1}{N} \sum_{n=1}^N [Y_n^2] \quad (4.4)$$

4.6.5 Standardization to Unit Standard Deviation

This operation is performed by dividing every amplitude Y_n by the value σ_y obtained from the equation (4.4).

$$Z_n = \frac{Y_n}{\sigma_y} \quad (4.6)$$

This standardization is convenient for fixed point computer calculation. The data Z has zero mean value and the standard deviation is equal to unity.

4.6.6 The Computation of the Probability Functions

To obtain a digital estimate of the probability density functions, it is necessary to divide the amplitude interval, say AB , into an appropriate number k of amplitude-class-intervals. Then by tabulating the number of observations, the percentage of data in each class interval can be found. This will give the probability density histogram of the signal.

An estimate of the probability distribution is then obtained by accumulating the numbers of observations in the domain from $-\infty$ to Z . Consider N_i the number of observations in each amplitude class interval. This procedure will sort N values of Z_n such that:

$$N = \sum_i N_i \quad (4.7)$$

The sequence of N_i can be used to produce the values of $p(z)$ and $P(z)$.

i) The hystogram of Z is the sequence of the successive

values of N_i .

ii) The sample percent of data is defined as:

$$P_{i,Z} = \frac{N_i}{N} \times 100 \% \quad (4.8)$$

iii) The sequence of sample probability density functions p_i , measured as percentage and defined at the mid-point of the i -th class interval:

$$P_{i,Z} = \frac{P_{i,Z}}{c} = \frac{N_i}{N} \cdot \frac{k}{(A-B)} \times 100, \quad i=1, \dots, k \quad (4.9)$$

where $c = \frac{k}{(A-B)}$ is the size of the amplitude class interval.

iv) The sequence of sample probability distributions $P(j)$ defined at the class interval end points:

$$P_Z(j) = \text{Prob} [-\infty \leq Z < d_j] \quad (4.10)$$

$$P_Z(j) = \sum_{i=0}^j P_{iZ} = c \sum_{i=0}^j P_{iZ}, \quad j=0, \dots, k+1 \quad (4.11)$$

These algorithms are used in the computer program annexed.

4.6.7 The Parameter Selection

The parameters should be selected for the computation of the probability density function. They are the amplitude interval (A,B) and the number k of amplitude class intervals. Since the probability function does not give information regarding the lengthwise description of the surface tex-

ture, it is not possible to correlate closely the sampling rate to these probabilities.

For the computation of the autocorrelation and of the spectral density functions however, the following parameters should be pre-selected [29] :

- i) The sampling rate and implicitly the Nyquist frequency;
- ii) the length of the record N ;
- iii) the bandwidth, B_e ;
- iv) the maximum number of autocorrelation lag, m .

To help the proper selection of these parameters, the following equations (Bendat [29]) are used:

$$h = \frac{L}{N} \cdot \frac{1}{v} \quad (4.12)$$

$$f_c = \frac{1}{2h} \quad (4.13)$$

$$m = N \cdot \epsilon^2 \quad (4.14)$$

$$B_e = \frac{1}{mh} \quad (4.15)$$

where

v = the speed of the stylus

f_c = the Nyquist frequency

h = the sampling time in seconds

m = the maximum autocorrelation

lag number

B_e = the bandwidth

N = the maximum number of data points

ϵ = the normalized standardized error
desired for the calculation of
the spectral density

L = the real length of the record

To start an experiment, 6 parameters have to be pre-selected and for the experiments annexed, the following parameters were chosen: A , B , k , h , N and ϵ . This is an operational selection where the sampling time h is selected in a liberal way, according to the hardware and software facilities. The value of ϵ is selected depending upon the accuracy required in the computation of the spectral density.

4.6.8 The Computation of the Autocorrelation Function

After the maximum lag number m is selected, the sample autocorrelation function of the digitized record Z_n , that is taken to be stationary with zero mean value, can be computed for any displacement λ in origin of the record. This shift λ may be represented by a lag number r , then:

$$R_{r,Z} = \frac{1}{N-r} \sum_{n=1}^N Z_n Z_{(n+r)} \quad (4.16)$$

where r is the lag number varying from zero to m . Recalling the equation (4.14), it may be seen that for a small

statistical uncertainty ϵ in the estimation of the spectral density, one should select m much smaller than N . On the other hand, a high resolution, which means a small value for the bandwidth B_e , will result in a large value for m . Thus, a compromise is necessary in order to achieve simultaneously a reasonable bandwidth and a reasonable statistical error ϵ .

The autocorrelation function can take positive and negative values and the following three conditions hold:

$$-1 \leq \frac{R_r}{R_0} \leq +1 \quad (4.16)$$

$$R(0) = \sigma^2$$

$$R(+r) = R(-r)$$

where R_0 is the autocorrelation at zero lag value and R_r is the autocorrelation at lag r .

4.6.9 The Spectral Density Function

For a stationary record Y_n with a zero mean value, a raw estimate of the spectral density function $G(f)$, for any arbitrary value of f from zero to f_c is defined by the following formula:

$$G(f) = 2h \left\{ R_0 + 2 \sum_{r=1}^{m-1} R_r \cos\left(\frac{\pi r t}{f_c}\right) + R_m \cos\left(\frac{\pi m f}{f_c}\right) \right\} \quad (4.17)$$

where h is the sampling time

R_r is the estimate of the autocorrelation for lag r

m is the maximum autocorrelation lag number
 f_c is the Nyquist frequency

The formula (4.17) gives $G(f)$ as a continuous function of f . For the digital computation a more convenient form of this equation is given by

$$f = \frac{k f_c}{m} = \frac{k}{2mh} \quad (4.18)$$

$$G_k = G\left(\frac{k f_c}{m}\right) \quad (4.19)$$

$$G_k = 2h \left\{ R_0 + 2 \sum_{r=1}^{m-1} R_r \cos\left(\frac{\pi r k}{m}\right) + (-1)^k R_m \right\} \quad (4.20)$$

$k = 0, 1, \dots, m$

where k is called the harmonic number. The value $k = 0$ represents the zero frequency or a straight line for z .

A convenient criterion which is based on all the $(m+1)$ estimates of G_k , is used in the computer program as a check on G_k .

$$\left| R(0) - \frac{1}{2mh} \left[.5G_0 + \sum_{k=1}^{m-1} G_k + .5G_m \right] \right| \leq \xi \quad (4.21)$$

where ξ must tend to zero as the sampling rate is increased.

A final smooth estimate of G_k may now be obtained by a procedure of frequency smoothing called "Hanning" [30]. Let S_k represent the "smooth" estimate at harmonic k . Then at the $(m+1)$ frequencies $f = \frac{k f_c}{m}$, where $k = 0, \dots, m$, one obtains:

$$S_0 = 0.5 G_0 + 0.5 G_1$$

$$S_k = 0.25 G_{k-1} + 0.5 G_k + 0.25 G_{k+1} \quad (4.22)$$

$$S_m = 0.5 G_{m-1} + 0.5 G_m$$

Smoothing is necessary since the raw estimate given by equations (4.18) to (4.20) is an inefficient estimate of the true spectral density. To be specific the error in these estimates does not decrease with increased record length or sample size. This leads to the requirement of smoothing the periodogram or equivalent, weighing the correlation function non-uniformly. The "Hanning" method of smoothing is simple to use on a digital computer and is connected with the experimental data. It may be mentioned that the area enclosed between the spectral density plot contain two ordinates f_i and f_j and the frequency axis, which represent the mean square value of the harmonic components situated in this bandwidth.

4.7 An Algorithm for Digital Filtering

In the process of data analysis it is sometimes necessary to eliminate the long wave harmonic components of the surface texture. This process is intended to eliminate waviness and to retain only the roughness of the surface irregularity. A digital filter on the basis of the algorithm presented below produces a sharp cut-off opposed to the analog filtering

produced by the electronic filters incorporated in the Talysurf 4 electronic unit.

The procedure of digital filtering consists of representing the surface texture function as a Fourier Series and eliminating the appropriate frequencies in this Fourier expansion.

Considering the speed v of the travelling stylus of the Talysurf 4, and considering any harmonic component (of frequency f and wavelength λ) present in the function Y , it is evident that the following relation holds:

$$f \left[\frac{\text{cycles}}{\text{sec}} \right] \cdot \lambda \left[\frac{\text{mm}}{\text{cycles}} \right] = v \left[\frac{\text{mm}}{\text{sec}} \right] \quad (4.23)$$

or

$$f = \frac{v}{\lambda} \quad \text{and} \quad \lambda = \frac{v}{f} \quad (4.24)$$

Bearing in mind the equation (4.24), for mathematical convenience, the following equations are written in the time domain and thus, the independent variables become t , namely

$$Y = Y(t)$$

Let us assume a surface texture record $Y = Y(t)$ of a finite period T_p , where T_p denotes the fundamental period in the data. The length of the record T_r is selected to be equal to T_p and then $T_r = T_p$. It is further assumed that the record Y (or the record Z which is linearly related to Y) is sampled at an even number of points N , equally spaced at h seconds apart. According to the equation (4.13)

the Nyquist frequency $f_c = \frac{1}{2h}$. If one considers the first point in the record Y to have a zero amplitude, it is then possible to express the finite version of the Fourier series which will pass through these N data values. For any point t in the interval from zero to T_p one obtains:

$$Y(t) = A_0 + \sum_{q=1}^{.5N} A_q \cos \left[\frac{2\pi q t}{T_p} \right] + \sum_{q=1}^{.5N-1} B_q \sin \left[\frac{2\pi q t}{T_p} \right] \quad (4.23)$$

The coefficients A and B being further defined. At the N discrete sampling points, the values of the amplitudes may easily be computed by putting $t = nh$ where h is the sampling time and n varies from one to N . At these particular points, the Fourier expansion becomes:

$$Y_n = Y(nh) = A_0 + \sum_{q=1}^{.5N} A_q \cos \left[\frac{2\pi n q}{N} \right] + \sum_{q=1}^{.5N-1} B_q \sin \left[\frac{2\pi n q}{N} \right] \quad (4.24)$$

where

$$A_0 = \frac{1}{N} \sum_{n=1}^N Y_n = 0 \quad (4.25)$$

$$A_q = \frac{2}{N} \sum_{n=1}^N Y_n \cos \frac{2\pi q n}{N} ; q = 1, 2, \dots, (.5N-1) \quad (4.26)$$

$$A_{0.5N} = \frac{1}{N} \sum_{n=1}^N Y_n \cos n\pi \quad (4.27)$$

$$B_q = \frac{2}{N} \sum_{n=1}^N Y_n \sin \frac{2\pi q n}{N} ; q = 1, 2, \dots, (.5N-1) \quad (4.28)$$

It should be noted that A_0 is zero because Y is the normalized version of U . The terms A_q are the finite

analogues of Fourier cosine coefficients, B_q are the finite analogues of the Fourier sine coefficients. For large N , the digital computation requirements for determining A_q and B_q can become quite extensive.

To produce a digital filtering with respect to the low frequency components, it is necessary to simply eliminate the corresponding components from the Fourier expansion and then the filtered signal is given by:

$$Y_f = Y_n - [A_0 + \sum_{q=1}^j A_q \cos(\frac{2\pi qn}{N}) + \sum_{q=1}^j B_q \sin(\frac{2\pi qn}{N})] \quad (4.29)$$

where A_0 , A_q and B_q have the same significance as before.

The determination of the number of terms to be filtered is straight forward:

$$\frac{q}{Nh} \leq f_{\text{min. residual}} \quad (4.30)$$

$$f_{\text{min}} = \frac{v}{\lambda_{\text{max}}}$$

$$\frac{q}{N \cdot h} \leq \frac{v}{\lambda_{\text{max residual}}}$$

and

$$q \leq \frac{v \cdot N \cdot h}{\lambda_{\text{max.res.}}} \quad (4.31)$$

This algorithm which produces a sharp cut-off should be utilized in the cases where a low frequency component is suspected to exist. This may occur most probably in such

manufacturing processes as turning, milling, etc. For the experiments presented in the appendices, it was not necessary to produce this kind of filtering.

FIGURE 4.1
BLOCK DIAGRAM OF TALYSURF 4

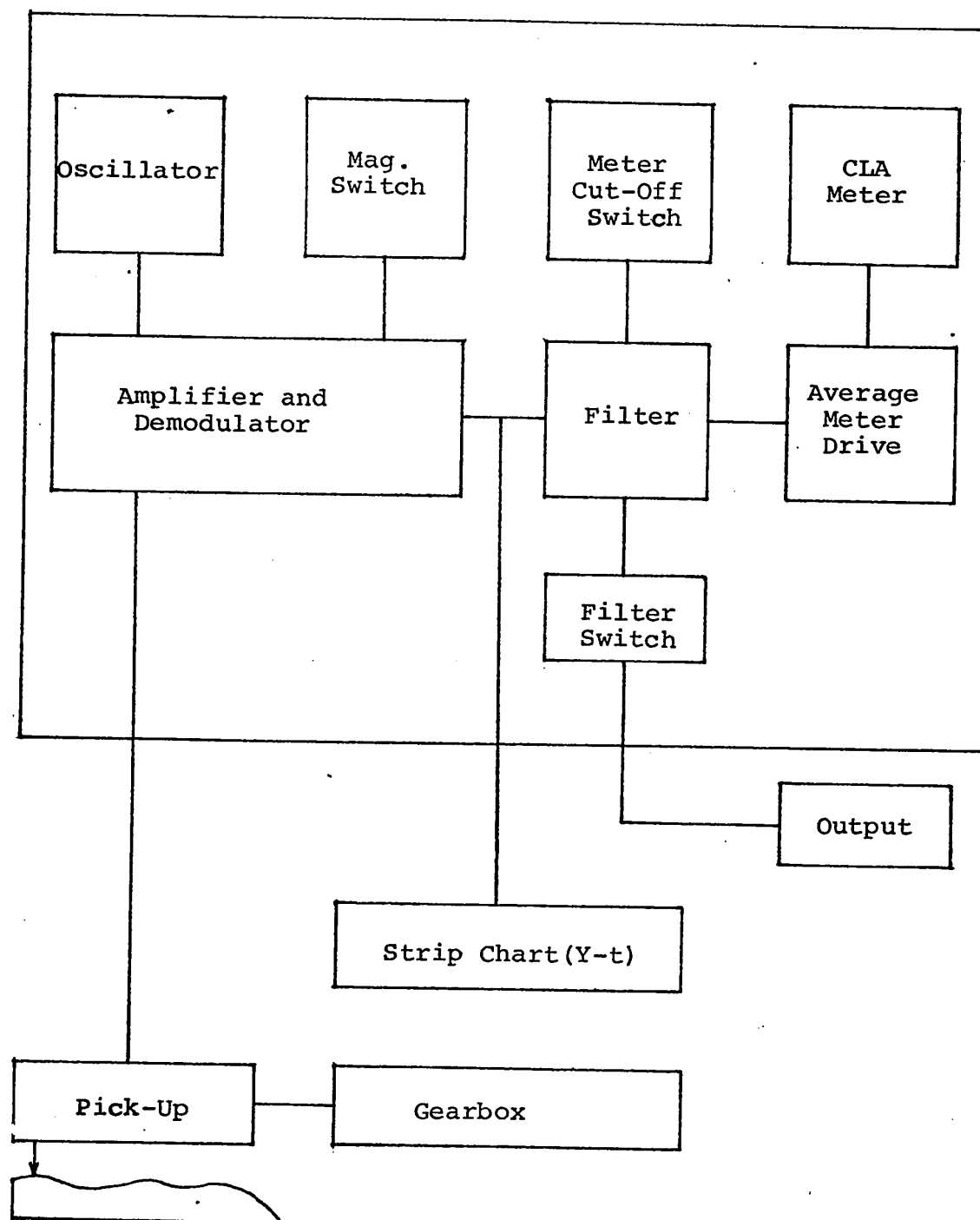


FIG. 4.1 Block Diagram of Talysurf 4^[31]

FIGURE 4.2
SURFACE TEXTURE SAMPLES PRODUCED BY THE TALY-
SURF 4 STRIP CHART RECORDER (GRINDING OPERATION)

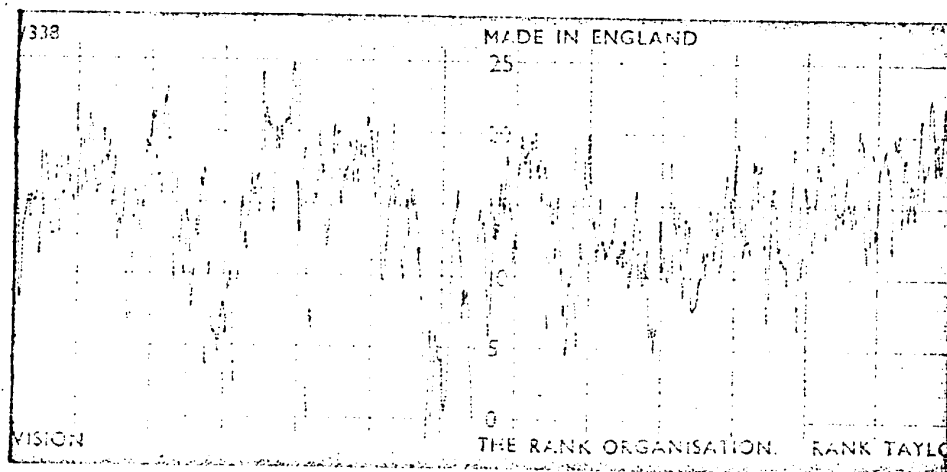


FIG. 4.2 Surface Texture Samples Produced by
the Talysurf 4 Strip Chart Recorder
(Grinding Operation)

FIGURE 4.3

CHART SHOWING THE EFFECT OF THE CUT-OFF
ON THE CLA-VALUE COMPUTED ON THE TALY-
SURF 4

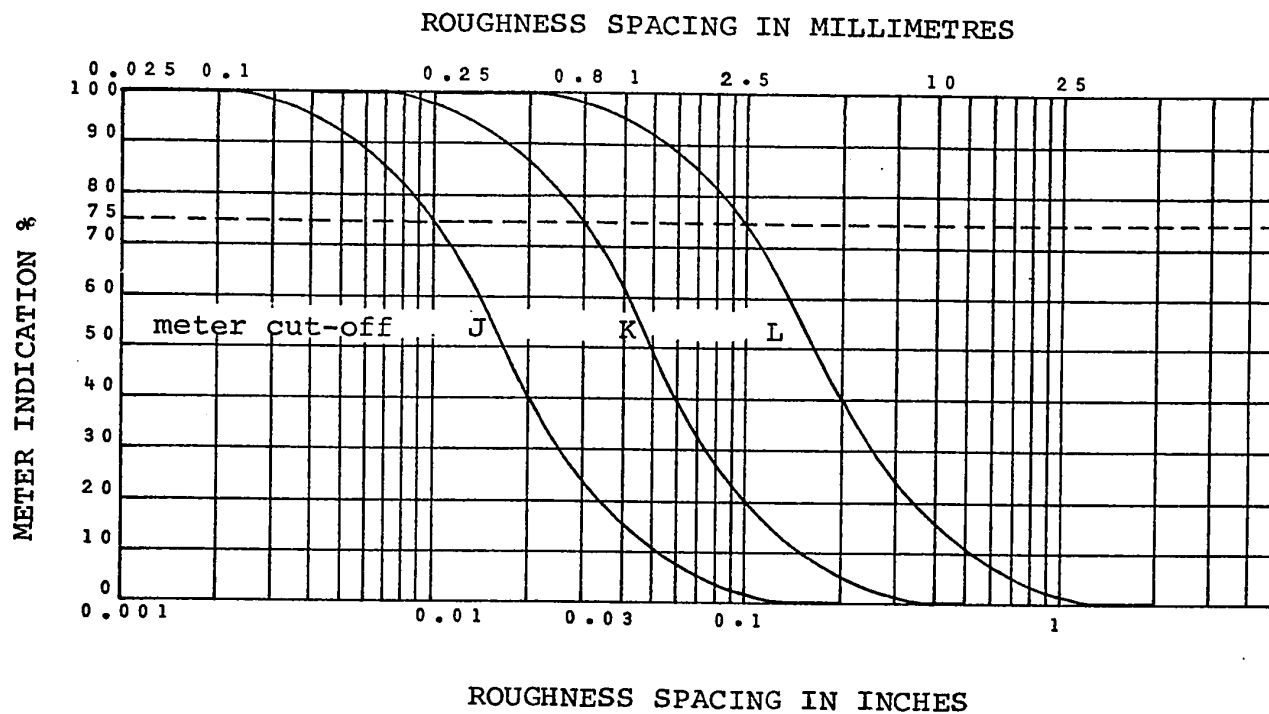


FIG. 4.3 Chart Showing the Effect of the Cut-Off
on the CLA-Value Computed on the Taly-
Surf 4 [31]

FIGURE 4.4

VALUES OF THE MAGNIFICATION FACTORS FOR DIFFER-
ENT MAGNIFICATION SWITCH POSITIONS FOR THE TALY-
SURF 4

BRITISH

METRIC

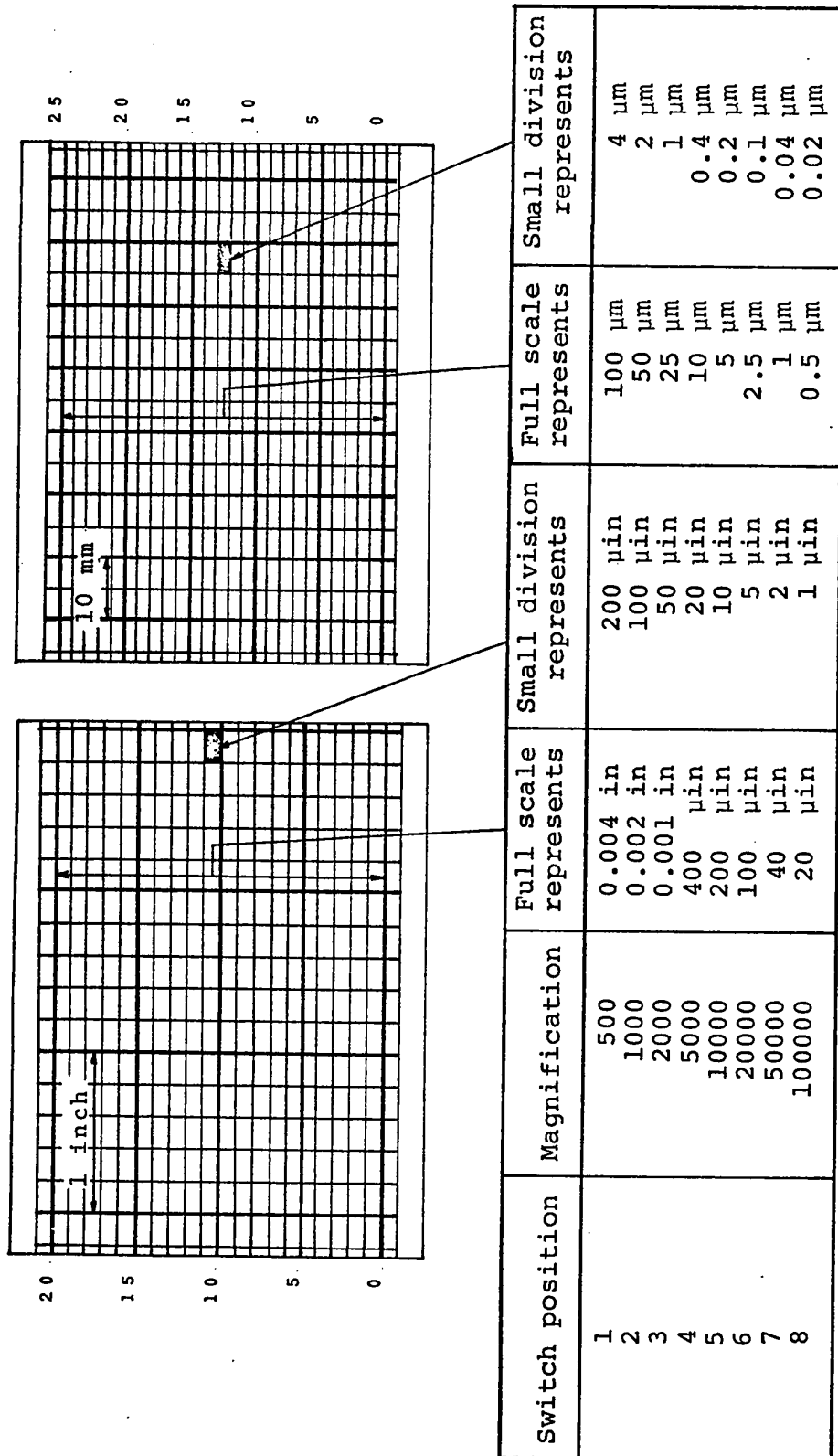


FIG. 4.4 Values of the Magnification Factors for Different Magnification Switch Positions for the Talysurf 4 [31]

FIGURE 4.5

**THE MEASURING AND THE DATA PROCESSING SYSTEM
UTILIZED FOR THE ANNEXED EXPERIMENTS**

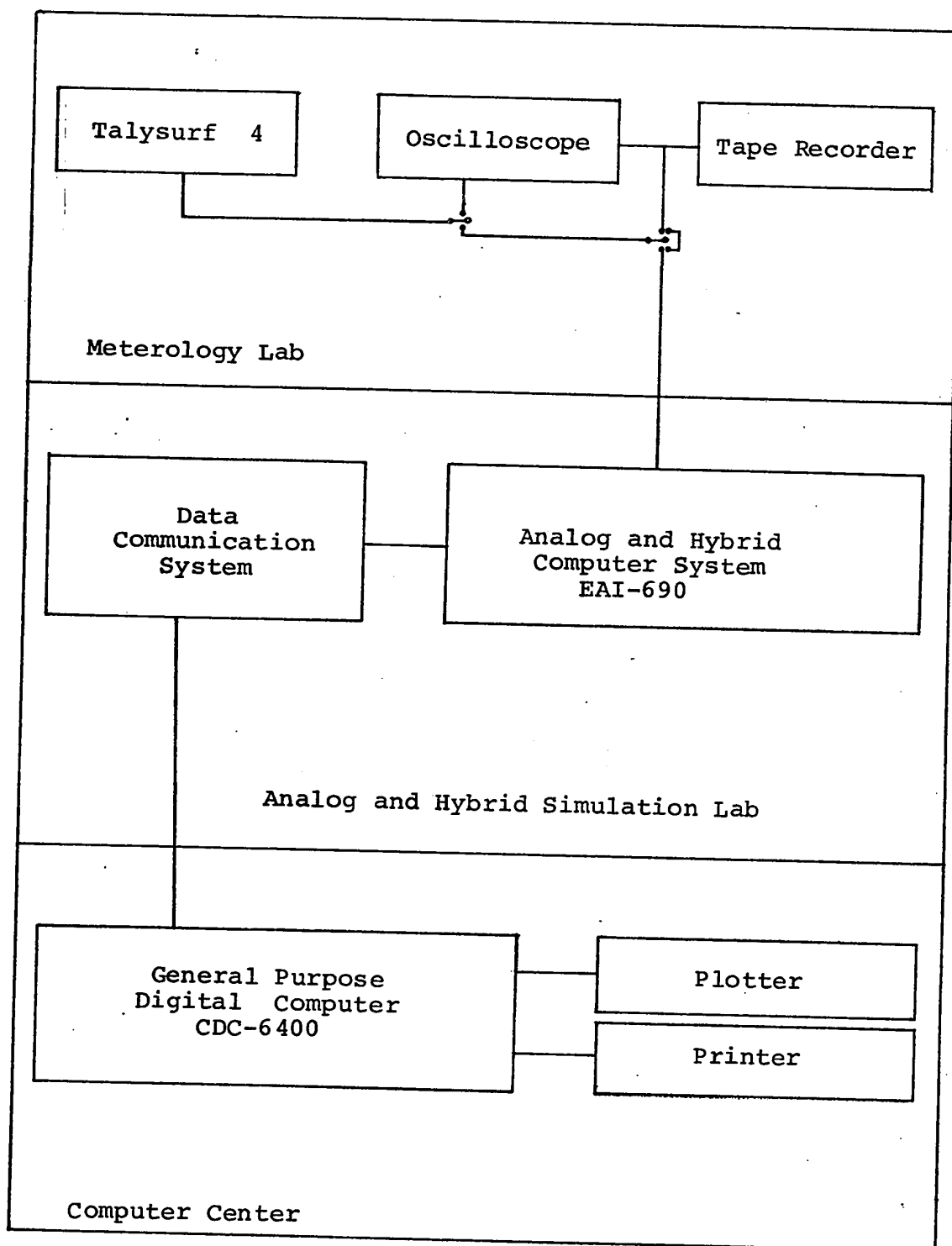


FIG. 4.5 The Measuring and the Data Processing System Utilized for the Annexed Experiments

CHAPTER V

EXPERIMENTAL RESULTS AND DISCUSSIONS

CHAPTER V

EXPERIMENTAL RESULTS AND DISCUSSIONS

5.1 Introduction

In support of the analytical considerations presented in Chapters 3 and 4, a number of experimental observations of surface texture samples were made and selected results are presented in the appendices. They consist of the output obtained from the computerized analysis of the surface texture in terms of plots of the stochastic functions described in Chapter 3.

5.2 Parameter Selection for Experiments

For the purpose of the experimental work, a sample surface produced by grinding a block of C1020 was utilized. After producing its "local" characterization by means of a strip chart recording of the amplitude and the CLA-value produced by the Talysurf 4, the different stochastic functions are computed for a complete statistical characterization of this sample surface. In brief, the experimental procedure consists of recording the signal that is collected at the output of the Talysurf 4-measuring device, and converting this analog signal into a digital form. Then, the digital data-points are processed in order to generate the stochastic descriptors. To achieve this, a number of parameters such as the sampling rate, the length of the record, the maximum value of the autocorrelation lag number,

amplitude domain, etc., are selected according to the relationships presented in equations (4.12) to (4.15).

Since the stochastic characterization of the surface depends largely on the choice of the parameters such as sampling rate, length of the record, the statistical error for the spectral density, etc., proper care must be exercised in the selection of these parameters for any particular experiment.

The following observations are relevant to the experimental work presented in this thesis.

5.2.1 The Sampling Process

Let h denote the time interval between any two successive readings in the process of digitizing the analog sample $U(\ell)$ into its equivalent discrete version U_n . Using the formula (4.12), the value of the maximum frequency may be computed and is useful for a frequency analysis of the sample. Because of the reverse relationship between the Nyquist frequency f_0 , and the sampling time h , reductions in h will successively enlarge the observable frequency domain. If the value of h is limited by the hardware equipment, it is not always useful just to select the highest possible sampling rate $\frac{1}{h}$, since data may not exist beyond certain frequency domains. This may entail extensive data processing costs and laborious analysis of the results that are not warranted for a particular case.

For the present experiment, a sampling time of 0.00164

and of 0.00328 seconds is utilized. This produces an observable frequency domain of:

$$f_o^{(1)} = \frac{1}{2h_1} = \frac{1}{2 \times 0.00164} = 304.8 \text{ cps}$$

$$f_o^{(2)} = \frac{1}{2h_2} = \frac{1}{2 \times 0.00328} = 152.4 \text{ cps}$$

Using the relationship which relates the frequency f , the wave length λ and the speed v of the travelling stylus of the Talysurf 4:

$$f \left[\frac{\text{cycles}}{\text{second}} \right] \cdot \lambda \left[\frac{\text{mm}}{\text{cycle}} \right] = v$$

it is possible to compute the wave length λ corresponding to the frequency f_o previously computed. Since the value of v utilized in the experiments is equal to 91.4 cm/min, we have

$$\lambda_1 = \frac{v}{f_o^{(1)}} = \frac{91.4/60}{304.8} = 0.005 \text{ mm}$$

$$\lambda_2 = \frac{v}{f_o^{(2)}} = \frac{91.4/60}{152.4} = 0.010 \text{ mm}$$

These represent the minimum wave lengths of the surface texture corresponding to the sampling time selected.

5.2.2 The Size of the Sampled Data

If the length of the travelling path L (in mm), is known, and if the travelling stylus has a speed v , then the total number of points N corresponding to the sampling

time h is:

$$N = \frac{L}{v} \frac{1}{h} = \frac{10}{91.4/60} \frac{1}{0.00164}$$

$$N = 4000 \text{ data points.}$$

5.2.3 The Maximum Autocorrelation Lag Number m

Bendat [29] presents a number of criteria for establishing the error in the computation of the stochastic functions. The normalized standard error ϵ for the spectral density is related to the maximum autocorrelation lag number m and to the total number N of data points by

$$m = N \cdot \epsilon^2$$

This particular criteria is selected here to determine the value of m , since it was previously established that the spectral density function gives the best surface description. For a good surface representation, it is suggested that the maximum value of ϵ does not exceed 0.1. The value of the parameters utilized for the experimental results annexed are given in Table 5.1. For smaller sample length, say $N = 320$ and $N = 100$ which correspond to the Talysurf 4 cut-off wavelengths of 0.8 mm and 0.25 mm respectively, the corresponding values of ϵ are 0.35 and 0.63 which are much higher than 0.1 and hence, are not acceptable.

Finally, the bandwidth B_e is obtained from the

formula

$$B_e = \frac{1}{mh} = \frac{1}{40 \times 0.00164} = 15.25 \text{ cps}$$

For easy comparison between the different spectral density plots obtained from the experiment, the same bandwidth is utilized in all the different sample lengths.

A resumé of the values of the parameters selected for the experimental results B,C,D,E is given in Table 5.1.

5.3 Experimental Results

The selected experimental results which are presented in the appendices are for a particular manufacturing process, namely grinding operations. A block of mild steel C1020 finished by grinding is utilized and the surface texture function, which is obtained is an electrical output from the Talysurf 4. Experiments were conducted for different parameter combinations, as detailed in Table 5.1. In the first case reported in Appendix B, the full travelling length of the travelling stylus of the Talysurf 4 is sampled into 4000 data points and analyzed. The sampling time h is taken as 0.00164 seconds per point, and the autocorrelogram lag number m is equal to 40. According to equation (4.11), the corresponding statistical error ϵ , for the spectral density is equal to 0.1. For the second experiment given in Appendix C, a reduction in the length of the travelling stylus to half stroke, or 5 mm, is utilized. For

TABLE 5.1
The Values of the Parameters Utilized in the Experiments Presented in the Appendices

Appendices	L	N	v	$h \times 10^{-3}$	ϵ	f_o	m	b_e	λ
	mm	points	$\frac{\text{mm}}{\text{min}}$	$\frac{\text{sec}}{\text{point}}$	#	cps	max	cps	mm
B	10.0	4000	91.4	1.64	0.10	304.8	40	15.25	.005
C	5.0	2000	91.4	1.64	0.14	304.8	40	15.25	.005
D	2.5	1000	91.4	1.64	0.20	304.8	40	15.25	.005
E	10.0	2000	91.4	3.28	0.10	152.4	20	15.25	.010

the same value for h , the number of data points N decreases from 4000 to 2000, the value of m is 40 and the statistical error ϵ increased to .14. In the third experiment, presented in Appendix D, the length of the surface sample was further reduced by half, making the length L to be equal to 2.5 mm. The same sampling time h was used with the value of m equal to 40, the value of ϵ being 0.2. This error may be considered still acceptable, even though it is recommended that ϵ be within 0.1. In the last experiment in Appendix E, a different sampling time $h = 0.00328$ was utilized for comparison. In all the experiments, the parameters were selected so as to result in the same value for the bandwidth, B_e in the spectral density computation. This facilitates one-to-one comparison of the final spectral density plots. The experimental results presented in the Appendices, B, C, D and E, contain the evaluation of stochastic functions in discrete form and are presented in Tables, as well as continuous plots.

5.4 Discussion of Results

5.4.1 Probability Density and Distribution Function

The probability density functions obtained have a bell-shape. This result is expected to occur, since it is actually recognized that manufacturing processes such as grinding, lapping, electrochemical processing, etc., produce surface texture irregularities that are normally distributed. Visual

inspection of all these functions obtained in the various experiments reveals that they are almost identical, lying within ten percent of each other. This provides a good verification of the assumption of stationarity of the process as far as the amplitudes distribution is concerned. The probability density function is not affected by a shift of the origin of the sample length. The smallest record analyzed is the one associated with Experiment D and it may then be taken as representative for the entire process. However, it is necessary to ensure a minimum value for the sample length in order to observe the property of stationarity in order to have a meaningful conclusion.

The probability distribution P contains the same information as the probability density function but in a different format. A Gaussian distribution as well as the closeness of the different functions P in the successive experiments may be observed here also. It might be remarked that the functions p and P are computed for a normalized and standardized record. The two statistical descriptions p and P have one limitation in that no information regarding the lengthwise surface texture description is obtainable from these records.

In the process of transmission of the digitized function by using a telephone data communication line, some technical difficulties were encountered. It was established that transmission errors for the data occurred in the region from

the data point 2300 to approximately the data point 2600. The presence of limited numbers of "abnormal" amplitudes did not affect the probability density and the probability distributions. However, their presence may have influenced to a certain extent, the autocorrelation function calculations.

5.4.2 The Autocorrelation Function

The autocorrelation function of the first experiment B_1 is a smooth curve, starting at the origin with the value one (1), and dropping smoothly to a value of 0.1, with a lag number 40. The shape of the autocorrelogram simply excludes the possibility of having a dominant frequency in the original data. The values of the amplitudes located in the region where transmission errors were reported are utilized in the computation of the functions presented in Appendix B.

By successively reducing the length of the record to half, i.e., to 2000 data points for the Experiment C, and to 1000 data points for Experiment D, the errors arising due to transmission difficulties are eliminated, to a certain extent. This explains then the difference in the shape of the autocorrelograms B and C, or B and D, and furthermore, it also justifies the similarity between the autocorrelation plots C and D. These take a zero value in the vicinity of the lag number $n = 22$. It is then possible to compute the correlation length which corresponds to this lag number

value. Denoting the correlation length by C_n :

$$C_n = v.h.n = (91.4/60) \quad 0.00164 \quad 22$$

$$C_n = 0.055 \text{ mm}$$

This means that the amplitudes of the record separated by more than a distance C_n , in this case 0.55 mm, are completely uncorrelated one to each other. To be able to observe the presence of the harmonic components (noise) superimposed on the random function (see also Fig. 3.4) and to approximately distinguish between a narrow band and a wide band harmonic noise, it is usually recommendable to consider for the stochastic analysis a sample length which infeds from 3 to 10 times the correlation length value. In the case of the experiment D, the sample length utilized is 2.5 mm and this corresponds to 43 times the correlation length.

The fact that the autocorrelograms observed in two different experiments conducted over different sample lengths, as shown in Table 5.1, have their corresponding zero value in the same region, denotes that the functions retain the basic characteristics and that they are not affected by a change of the length axis. This is an additional test for stationarity. Comparing the variances of the functions y corresponding to each experiment, it may be observed that their values are within six percent of each other. More details are given in Table 5.2.

TABLE 5.2
The Test of Weak Stationarity for the Surface
Texture Analyzed

Experi- ment	L	N	$h \times 10^{-3}$	\bar{Y}	σ_z	σ_y	Error compared to B
	mm	# points	$\frac{\text{sec}}{\text{point}}$	μm	#	μm	
B	10.0	4000	1.64	0	1.0	0.431	0.0
C	5.0	2000	1.64	0	1.0	0.433	0.006
D	2.5	1000	1.64	0	1.0	0.407	0.055

NOTE:

An error of ± 6 percent in the value of the standard deviations of the normalized records of a random process is acceptable for the test of weak stationarity.

5.4.3. The Spectral Density Function

The spectral density functions are very similar in their shapes and decrease rapidly and smoothly to zero from a peak at the origin. Most of the harmonic components are concentrated in the region from zero frequency up to harmonic number 20 which corresponds to a wave length of 0.009 mm. The original function is of a wide band type and except for the values in the vicinity of the origin, there is no evidence of predominant harmonics. The frequency structure is typical for this type of manufacturing process as opposed, for instance, to the spectral density of a turned part where a sharp peak may be present at a frequency corresponding to that of the axial feed.

The frequency range is determined by the value of the sampling time h and for the current experiments which are presented in the appendices. The Nyquist frequency is equal to 304.8 and 152.4 cps. This reduction of the frequency domain is due to the increase of the sampling time by a factor of 2 in experiment E. All the spectral plots are characterized by the same bandwidth and consequently, in the case of experiment E, the frequency domain in millimeters, is equal to half the value of the frequency domain in the previous experiment.

To understand the purpose of the spectral density curves, consider the area enclosed in a bandwidth located at the frequency number ten, Fig. B-8 in Appendix B. This

area represents the mean square value of those harmonic components in the original data, having the frequency in this range. Denoting this mean square value by σ_{10}^2 , then:

$$\sigma_{10}^2 = \sigma_Y^2 \text{ Area } [f_{10}; B_e]$$

$$\sigma_{10}^2 = (4.3 \text{ } 10^{-1}) (2.577 \text{ } 10^{-3}) 10.25$$

$$\sigma_{10}^2 = 0.01689 \text{ mm}^2$$

This property of the spectral density enables then the detailed analysis of harmonic components of frequencies from zero to f_0 providing a two-dimensional characterization of the surface texture in both the longitudinal and the axial directions.

The stochastic analysis which is produced in the appendices characterize the normalized and standardized function Z which is nondimensional. They are related to the appropriate descriptors of the function Y by the factors σ_Y^2 .

5.4.4 The Influence of the Sample Length and of the Talysurf Built-In Cut-Off

A specific test was performed to establish the correspondence between the measured values of the CLA - parameter as obtained from the CLA meter mounted on the Talysurf and the values of the CLA computed from the data points. The aim of this test is to determine whether the sampled data (4000 data points collected over a 10 mm sample length) is a

good representation of the continuous surface texture function. The second and more important objective of this experiment is to observe the effect of the built-in cut-off filtering device in Talysurf 4. These devices produce a progressive attenuation of the wave-length which is higher than the nominal selected cut-off, which for the Talysurf are equal to 2.5, 0.8 and 0.25 mm. A digital filter, as the one presented in Chapter V, produces a sharp cut-off, unless a weighting function is utilized. The experimental values presented in Table 5.3 show that the computed values of the CLA and those measured do not differ significantly. Since the CLA measured value for 2.5 mm, 0.8 mm and 0.25 mm cut-offs differ slightly one can conclude that all the wave-lengths are equal or smaller than 0.25 mm. The computed CLA values confirm this observation. However, it might be pointed out that if any correlation between the number of data points and the cut-off effect on the CLA evaluation is required, it should be based on much lesser data points than 100, so that an equivalent cut-off length smaller than 0.25 mm is obtained. This should be a matter of a future examination.

The frequency component-number corresponding to the wave-length of 0.25 mm is:

$$f = \frac{k}{2\pi h} = \frac{v}{\lambda}$$

$$k = \frac{2\pi h v}{\lambda} = \frac{2 \cdot 40 \cdot 0.00164 \cdot (91.4/60)}{0.25}$$

$$k = 0.8$$

5.5 Recommendation for Future Work

This thesis analyzes manufactured surfaces as a stochastic process and results are verified using extensive experimental observations. The mathematical concept utilized in this work is basic and hence, the results are reasonably valid in describing surface irregularities. There exists certain drawbacks in this work, as far as a thorough surface description is concerned. It may be possible to utilize extensions of these ideas to incorporate all the characteristic parameters required to define surfaces.

One possibility of providing lengthwise characteristics of surfaces is to calculate the probabilities of the excursion intervals of the surface signal. Such a method is employed by Sankar and Osman [23]. This method includes the advantages of relating the excursion duration probability at different percentages of CLA-values to mechanical properties such as bearing area curve, etc.

The method reported in this thesis and in other recent literature is not applicable to surfaces that do not possess stationary properties. Extension of the analysis to non-

TABLE 5.3

The Measured CLA_T , and the Computed CLA_C Values for Different Cut-Off Switch Positions and for Different Record Length L

N	L	Computed CLA_C		Measured CLA		$\frac{CLA_T}{CLA_C}$
		No. of Samples	Average CLA_C	Cut-Off Switch Position	Measured CLA	
# points	mm					
4000	10.00	1	0.3294	L	0.3575	1.074
2000	5.00	2	0.3310	K	0.3275	0.990
1000	2.50	4	0.3046	J	0.3237	0.950
300	0.80	11	0.3334	-	-	-
100	0.25	35	0.3137	-	-	-

stationary surface signals will provide means of discussing surfaces possessing large curvatures.

The present work can also be extended with certain modifications to determine roundness of surfaces and to specify surface finishes in manufacturing and production.

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APPENDIX A
THE COMPUTER PROGRAM AND DATA

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Figure

A-1 Flow chart for the surface texture analysis	A-2
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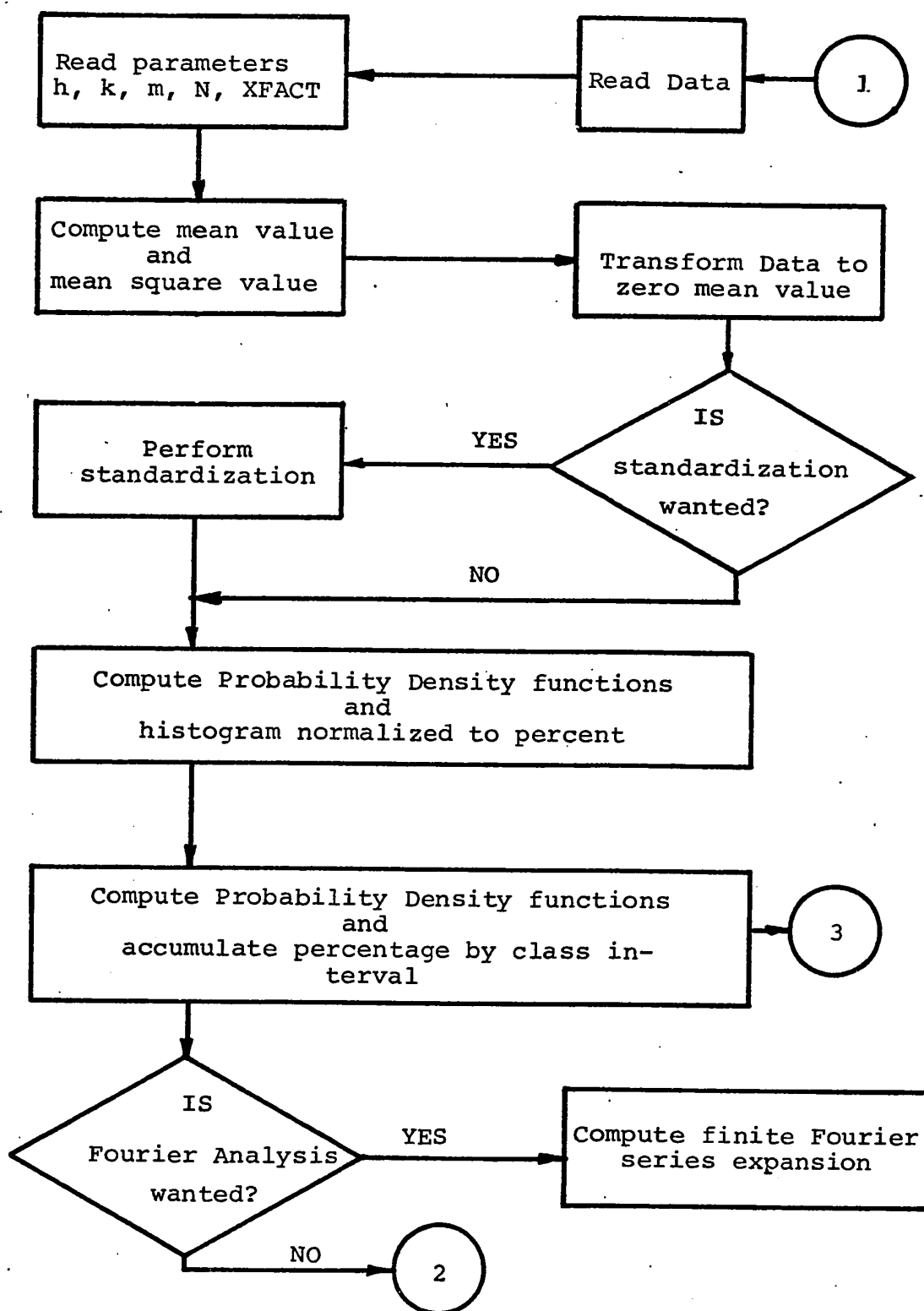
Table

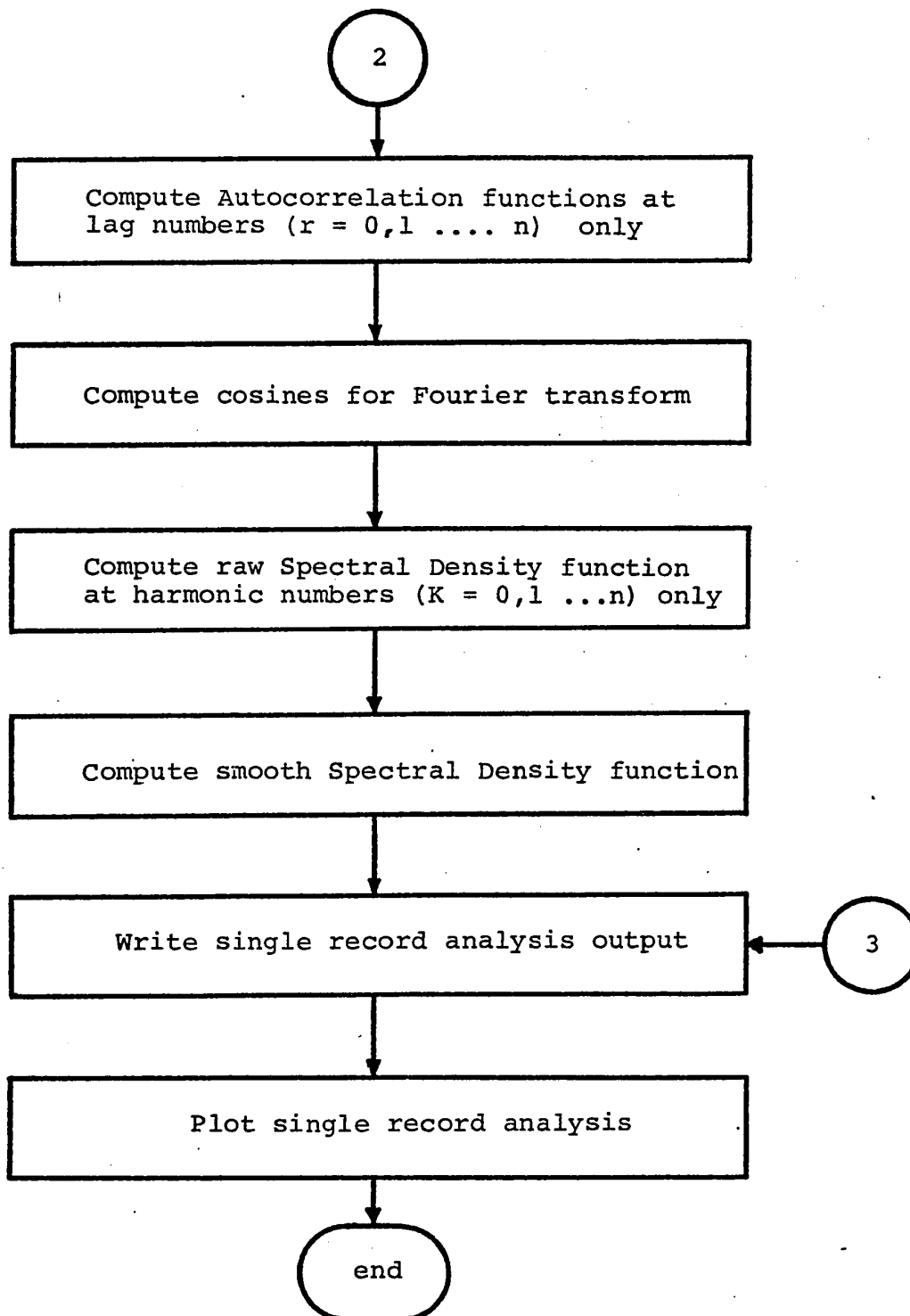
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A-3.2 The discrete function Z_n , 4000 data points.	A-21

FIGURE A-1

FLOW CHART FOR THE SURFACE TEXTURE ANALYSIS

FIG. A-1 FLOW CHART FOR THE SURFACE TEXTURE ANALYSIS





A-2 COMPUTER PROGRAM LISTING FOR THE STOCHASTIC
ANALYSIS OF THE SURFACE TEXTURE

A-2 COMPUTER PROGRAM LISTING FOR THE STOCHASTIC ANALYSIS OF THE SURFACE TEXTURE

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PROGRAM TS
COMMON IARRAYS(14000)
DIMENSION IARRAYS(7000)
EQUIVALENCE (IARRAYS,IARRAYS)
DIMENSION X(4000),PARAM(50),IHIST(100),PDENS(100),POIST(100),
1 PERCENT(100),GAUSS(100),R(400),G(400),GR(400),FR(400),W(400)
C
C SUBMITTED AS PARTIAL FULLFILMENT FOR THE MENG. DEGR.. SEPT 1970, T. SCHAIN.
C
C SET UP STORAGE ALLOCATION FOR ARRAYS IN COMMON, ALLOWING
C FOR ZERO SUBSCRIPTS.
EQUIVALENCE (X,IARRAYS(1)),(IHIST,IARRAYS(8003)),
1 (PDENS,IARRAYS(8203)),(POIST,IARRAYS(8403)),
2 (PERCENT,IARRAYS(8603)),(GAUSS,IARRAYS(8803)),
3 (R,IARRAYS(9003)),(G,IARRAYS(9203)),(GR,IARRAYS(10603)),
4 (FR,IARRAYS(11403)),(W,IARRAYS(12203))
COMMON/DATA/PARAM
DIMENSION IPARAM(100)
EQUIVALENCE (PARAM,IPARAM)
EQUIVALENCE (N,IPARAM(1)),(M,IPARAM(2)),(A,IPARAM(3)),
1 (R,IPARAM(5)),(K,IPARAM(7)),(IFSTAN,IPARAM(8)),
2 (H,IPARAM(9)),(IFPTRANS,IPARAM(11)),(ISETS,IPARAM(12)),
3 (XFACT,IPARAM(13)),(GAIN,IPARAM(15))
DATA (GAUSSCON=.3989422804)
C X=VECTOR OF DATA POINTS, OBTAINED BY A CALL TO THE USER -SUPPLIED
C SUBROUTINE DATA IN X WILL BE NORMALIZED AND IF REQUESTED,
C STANDARDISED TO ZERO STANDARD DEVIATION.
C
C A SUBROUTINE IS CALLED TO OBTAIN PARAMETERS AND OPTIONS.
C THE RESULT WILL BE A SEQUENCE OF VALUES AND FLAGS IN THE ARRAY PARAM.
CALL GET PARAM
C NOW THE DATA ARE OBTAINED.
C DATA WILL BE NORMALISED AND STANDARDISED OR ONLY NORMALISED.
NNN=1
5000 CALL DATA IN(X,ISWITCH,ISPEED)
C CONVERT MACHINE UNITS TO VOLT TO INCHES
XINCH=XINCH*XFACT/GAIN
DO 6000 I=1,N
X(I)=X(I)*XINCH
6000 CONTINUE
PRINT 5001
5001 FORMAT(1H1)
PRINT 5100,ISWITCH,ISPEED,N,K,M,H
5100 FORMAT(/# EXPERIMENT NO.##/# MATERIAL##/# TECHNOLOGY#/
1 # 1#/# 2#/# 3#/# 4#/# 5#/# 6#////
2 # MEASURING CONDITION##/# VERT.MAGN.SWITCH#,10X,I4/
3 # SPEED (H.MAGN)#,11X,I4/
4 ##/# PARAMETERS OF THE COMPUTATION#/
5 # NUMBER OF POINTS#, 9X,I4/# NO.OF CLASS INTERVAL#, 5X,I4/
6 # MAX.LAG NUMBER#,11X,I4,/# SAMPLING RATE#,13X,F7.5//)
PRINT 5001
PRINT 1,(X(I),I=1,N)
1 FORMAT(14H0SAMPLED DATA//(10E13.4))
AVG=SD=0.

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      XMIN=XMAX=X(1)
      DO 10 I=1,N
      XI=X(I)
      IF(XI-XMIN)2,8,4
    2  XMIN=XI
      GO TO 8
    4  IF(XI-XMAX)8,8,6
    6  XMAX=XI
    8  AVG=AVG+XI
   10  SD=SD+XI*XI
      XN=N
      XNINV=1./XN
      AVG=AVG*XNINV
      SD=SQRT((SD-XN*AVG*AVG)/(XN-1.))
      PRINT 1001,N,AVG,SD,XMIN,XMAX
1001 FORMAT(9H NUMBER=,I5/12H AVERAGE=,E12.5/
1  23H STANDARD DEVIATION=,E12.5/
2  12H MINIMUM=,E12.5/12H MAXIMUM=,E12.5//)
C) CHECK TO SEE IF STANDARDISATION IS WANTED.
   IF (IFSTAN) 40,40,20
C
C
C  NORMALISED AND STANDARDISED.
C
   20 SDINV=1./SD
   DO 30 I=1,N
   30 X(I)=(X(I)-AVG)*SDINV
      XMIN=(XMIN-AVG)*SDINV
      XMAX=(XMAX-AVG)*SDINV
      PRINT 31,XMIN,XMAX
   31 FORMAT(51H THE DATA HAVE BEEN NORMALISED AND STANDARDISED //
1  9H MINIMUM=,E12.5/9H MAXIMUM=,E12.5/
2  23H STANDARD DEVIATION=1.0//)
      GO TO 51
C
C
C  NORMALISED BUT NOT STANDARDISED.
C
   40 DO 50 I=1,N
   50 X(I)=X(I)-AVG
      XMIN=XMIN-AVG
      XMAX=XMAX-AVG
      PRINT 49,XMIN,XMAX,SD
   49 FORMAT(52H THE DATA HAVE BEEN NORMALISED BUT NOT STANDARDISED//,
1  9H MINIMUM=,E12.5/9H MAXIMUM=,E12.5/
2  20H STANDARD DEVIATION=,E12.5//)
C CHECK TO SEE IF PRINTING IS WANTED.
   51 IF (IFPTRANS) 59,59,52
   52 I1=1
      I10=10
   53 IF (I10-N) 55,55,54
   54 I10=N
   55 PRINT 56,(X(I),I=I1,I10)
   56 FORMAT (10E13.4)
      I1=I1+10

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      IF(I1-N) 57,57,59
57 I10=I10+10
   GO TO 53

C
C SELECT K=THE NO. OF CLASS INTERVAL.
C K WILL BE CONSIDERED TO BE AN INPUT PARAMETER.
C SELECT A AND B ( THE LOWER AND THE UPPER POINT ).
C COMPUTE THE HISTOGRAM.
59 K1=K+1
   DO 60 I=0,K1
60 IHIST(I)=0
   C=K
   A=-5.
   B= 5.
   C=(B-A)/C
   CINV=1./C
C THE METHOD IS TO ADD 1 TO IHIST(J) IF D(J-1).LE.X(I).LT.D(J)
C WHERE D(J) IS THE J-TH CLASS INTERVAL.
   DO 110 I=1,N
     XI=X(I)
     IF(XI-A) 70,100,80
70 IHIST(0)=IHIST(0)+1
   GO TO 110
80 IF (XI-B) 100,90,90
90 IHIST(K1)=IHIST(K1)+1
   GO TO 110
100 INDEX=(XI-A)*CINV+1.
    IHIST(INDEX)=IHIST(INDEX)+1
110 CONTINUE
C COMPUTE PERCENTAGES,PROB.DENSITY,AND PROB.DISTRIBUTION.
   ACCUM=0.
   DO 120 I=0,K1
     PERCENTI=IHIST(I)
     PERCENT(I)=PERCENTI*PERCENTI*XNINV
     PDENS(I)=PERCENTI*CINV
120 PDIST(I)=ACCUM+ACCUM*PERCENTI
C**INSERT PLOT ROUTINES.
C COMPUTE THE GAUSS DISTRIBUTION.
C ACCUMULATE THE QUANTITY FOR THE CHI-SQUARE TEST.
   DO 130 I=0,K1
C GAUSSCON=1./SQRT(2.*PI)=0.39894228040)
     CHISQ=0
     XI=I
     XI=A+C*(XI-.5)
     GAUSS(I)=GAUSSCON*EXP(-XI*XI*.5)
     TEMP=GAUSS1-PDENS(I)
     CHISQ=CHISQ+TEMP*TEMP/GAUSS1
130 CONTINUE
C PRINT THE AMPLITUDE STATISTICS.
   PRINT 131
131 FORMAT(27H1DISTRIBUTION OF AMPLITUDES.,7H0 CLASS,7X,5H RANGE,
1      11X,8HHIDPOINT,9X,9HHISTOGRAM,11X,7HPERCENT,12X,
2      11HPROB. DENS.,9X,12HPROB. DISTR.)
   R2=A

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      XI=A-.5*C
      PRINT 132,R2,XI,IHIST(0),PERCENT(0),PDENS(0),PDIST(0)
132  FORMAT(18H  0 -INFINITY TO,2F10.4,I15,3F20.4)
      DO133 I=1,K
      R1=R2
      R2=R2+C
      XI=XI+C
133  PRINT 134,I,R1,R2,XI,IHIST(I),PERCENT(I),PDENS(I),PDIST(I)
134  FORMAT(15,F10.4,3H TO,2F10.4,I15,3F20.4)
      R1=R2
      XI=XI+C
      PRINT 135,K1,R1,XI,IHIST(K1),PERCENT(K1),PDENS(K1),PDIST(K1)
135  FORMAT(15,F10.4,13H TO +INFINITY,F10.4,I15,3F20.4)
      PRINT 137,XMIN,XMAX,K,C,M,SN
137  FORMAT (//9H MINIMUM=E23.5/9H MAXIMUM=E23.5/
1      19H NO OF CLASS AMPL.=,I10 /21H CLASS INTERVAL SIZE=E12.5/
2      12H MAX.LAG NO=,I10 /20H STD.DEV.CORRECTION=E12.5//)
C**INSERT PLOT ROUTINES.
C  SELECT M THE MAX.LAG NO.
C  OPTIONS FOR M (READ IN M=.), (M=1/(BE*H)), (M=N/10.) OR (M=N*EPS**2).
C  BEGIN COMPUTATION OF THE AUTOCORRELATION FUNCTION.
C  THIS SECTION IS ABRIDGED FROM FROM ROUTINE AUTO PP 59-SSP MANUAL.
      DO 150 J=0,M
      NJ=N-J
      SUM=0.
      DO 140 I=1,NJ
      IJ=I+J
140  SUM=SUM+X(I)*X(IJ)
      FNJ=SUM
150  R(J)=SUM/FNJ
C  END OF AUTOCORRELATION SECTION.
C**INSERT PLOT ROUTINES.
C  BEGIN COMPUTATION OF THE FOURIER TRANSFORM.
C  (SLIGHTLY MODIFIED VERSION OF THE ROUTINE FORIT PP.275 SSP MANUAL.
C  C  COMPUTE COS(PI/M) AND SET UP OUTER RECURSION VARIABLES.
      C0=M
      C2=C0=COS(3.1415926536/C0)
      C=C1=1.
      J=0
      RM=R(M)
C  M+1 COEFFICIENTS ARE TO BE COMPUTED,GOING FROM J=0,TOJ=M.
C  RM ALTERNATELY EQUALS +R(M) AND -R(M),EACH SUM GOES FROM
C  I=M-1 TO I=U. U0,U1,ANDU2 ARE THE TERMS IN THE INNER REC.
160  U2=U1=0.
      I=M-1
      CC=C+C
170  U0=R(I)+CC*U1-U2
      U2=U1
      U1=U0
      I=I-1
      IF(I) 180,180,170
180  GR(J)=2.*H*(R(0)+2.*(C*U1-U2)+RM)
      IF(J=M) 190,190,200
190  C=2.*C1*C0-C2

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      C2=C1
      C1=C
      RM=RM
      J=J+1
      GO TO 160
C   INSERT PLOT ROUTINE
C   BEGIN SMOOTHING THE POWER SPECTRAL DENSITY.
200 GO=GR(0)
      G(0)=.5*(GR(1)+GO)
      M1=M-1
      DO 210 I=1,M1
      GM1=GO
      GO=GR(I)
210 G(I)=.25*(GM1+GR(I+1))+.5*GO
      G(M)=.5*(GM1+GR(M))
      DO 250 I=1,M
      FR(I)=I/(2*M*M)
      W(I)=91.4/(60.*FR(I))
250 CONTINUE
      PRINT 400
400 FORMAT (39H1AUTOCORRELATION AND SPECTRAL FUNCTIONS//
1      2X,7H LAG NO,5X,16H AUTOCORRELATION,5X,
2      21H RAW SPECTRAL DENSITY,5X,24H SMOOTH SPECTRAL DENSITY,5X,
3      11H FREQUENCY ,5X,12H WAVE LENGTH,5X,9H HARM.NO.//)
      PRINT 500,(J,R(J),GR(J),G(J),FR(J),W(J),J,J=0,M)
500 FORMAT(17,E21.4,E23.4,E28.4,E22.4,E17.4,I12)
C   CALCULATE THE MAXIMUM SPECTRAL DENSITY VALUE
      GMAX=ARRAYS(4901)
      DO 450 I=1,M
      GI=G(I)
      IF(GI-GMAX)450,450,440
440 GMAX=GI
450 CONTINUE
C   CALCULATE THE MINIMUM AUTOCORRELATION VALUE
      RMIN=R(1)
      DO 550 I=1,M
      RI=R(I)
      IF(RI-RMIN)470,550,550
470 RMIN=RI
550 CONTINUE
C   CHECK THE RAW SPECTRAL DENSITY
      SUM=0.
      MM=M-1
      DO 600 I=1,MM
      GGR=GR(I)
      SUM=SUM+GGR
600 CONTINUE
      TSTGR=.5/(H*M)*(.5*ARRAYS(5301)+SUM+.5*GR(M))
      TST=TSTGR-ARRAYS(4501)
      PRINT 700,TST
700 FORMAT (//22H TEST FOR RAW SP.DENS=,F13.9//)
      IF (IFSTAN ) 710,710,720
710 PRINT 730
730 FORMAT (35H NORMALISED BUT NOT STANDARDISED

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      GO TO 745
    720 PRINT 740
    740 FORMAT (35H NORMALISED AND STANDARDISED )
    745 PRINT 750,XMIN,XMAX,N,K,M,SD
    750 FORMAT (//9H MINIMUM=,E23.5/9H MAXIMUM=,E23.5/
      1      19H NO OF POINTS REC.=,I10 /19H NO OF CLASS AMPL.=,I10 /
      2      12H MAX.LAG NO=,I10 /20H STD.DEV.CORRECTION=,E12.5//)
    C  INSERT PLOT ROUTINES
    C  PLOT POINTS TO CHECK THE DIGITISED READING
    C  PLOT PDENS.
      CALL GRF(8203,K,-1.0,.0.,FACT)
    C  PLOT PDIST
      CALL GRF(8403,K,-1.0,.0.,FACT)
    C  PLOT AUTO
      CALL GRF(9003,M,0.0,.0.,RMIN,FACT)
    C  PLOT SP.DENS.
      CALL GRF(9803,M,1.0,04.0.,FACT)
      NNN=NNN+1
      IF(NNN=ISETS)5000,5000,7000
    7000 CALL ENDPLOT(.0,.0)
      STOP
      END

```

FORTRAN DIAGNOSTIC RESULTS FOR TS

NO ERRORS

TS P 02733 C 33260 D 00144

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SUBROUTINE GET PARAM
COMMON/DATA/PARAM
DIMENSION PARAM(50),IPARAM(100)
EQUIVALENCE (PARAM, IPARAM)
EQUIVALENCE (N,IPARAM(1)),(M,IPARAM(2)),(A,IPARAM(3)) ,
1      (R,IPARAM(5)),(K,IPARAM(7)),(IFSTAN,IPARAM(8)),
2      (H,IPARAM(9)),(IFPTRANS,IPARAM(11)),(ISETS,IPARAM(12)),
3      (XFACT,IPARAM(13)),(GAIN,IPARAM(15))
RETURN
END

```

FORTRAN DIAGNOSTIC RESULTS FOR GETPARAM

NO ERRORS

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GETPARAM  P  00021  C  00000  D  00144

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FORTRAN (3.2)/MASTER

05/10/71

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SUBROUTINE DATA IN(X,ISWTC,ISPEED)
COMMON/ DATA,PARAM
DIMENSION IPARAM(100)
EQUIVALENCE (PARAM,IPARAM)
EQUIVALENCE (N,IPARAM(1)),(M,IPARAM(2)),(A,IPARAM(3)),
1      (R,IPARAM(5)),(K,IPARAM(7)),(IFSTAN,IPARAM(8)),
2      (H,IPARAM(9)),(IFPTRANS,IPARAM(11)),(ISETS,IPARAM(12)),
3      (XFACT,IPARAM(13)),(GAIN,IPARAM(15))
DIMENSION X(1)
C PUNCH DATA VALUES FOR M,K,A,B,IFSTAN,IFPTRANS,H
C GAIN=TOTAL VOLTAGE GAIN AFTER THE FIRST OUTPUT
C MAGNIFICATION SWITCH 1....XFACT=50
C MAGNIFICATION SWITCH 2....XFACT=25
C MAGNIFICATION SWITCH 3....XFACT=12.5
C MAGNIFICATION SWITCH 4....XFACT=5.
C MAGNIFICATION SWITCH 5....XFACT=2.5
C MAGNIFICATION SWITCH 6....XFACT=1.25
C MAGNIFICATION SWITCH 7....XFACT=0.5
C MAGNIFICATION SWITCH 8....XFACT=0.25
      IFSTAN=1
      IFPTRANS=1
      ISET=1
      READ(60,100) ISWTC,ISPEED,N,K,M,H,XFACT,GAIN
100 FORMAT(5I5,3F8.5)
      READ(60,1) (X(I),I=1,N)
1 FORMAT(6(1X,F11.8))
      RETURN
END

```

FORTRAN DIAGNOSTIC RESULTS FOR DATAIN

NO ERRORS

DATAIN P 00132 C 00000 D 00144

FORTRAN (3.2)/MASTER

09/10/71

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      SUBROUTINE GRF(II,JJ,KK,XMAX,XMIN,FACT)
C   II DENOTES THE FIRST POSITION OF THE FUNCTION IN THE COMMON IARRAYS
C   JJ .EQ.K,OR.M AND DEFINES THE NO.OF POINTS TO BE PLOTTED
C   KK IS A PARAMETER WHICH SELECTS THE SUBR.AXISXY.
      COMMON IARRAYS(14000)
      DIMENSION ARRAYS(7000)
      EQUIVALENCE (ARRAYS,IARRAYS)
C   SET UP STORAGE ALLOCATION FOR ARRAYS IN COMMON ,ALLOWING
C   FOR ZERO SUBSCRIPTS.
      EQUIVALENCE (X,IARRAYS(1)),(IHIST,IARRAYS(8003)),
1      (PDENS,IARRAYS(8203)),(PDIST,IARRAYS(8403)),
2      (PERCENT,IARRAYS(8603)),(GAUSS,IARRAYS(8803)),
3      (P,IARRAYS(9003)),(G,IARRAYS(9203)),(GR,IARRAYS(10603)),
4      (FR,IARRAYS(11403)),(W,IARRAYS(12203))
      COMMON/DATA/PARAM
      DIMENSION PARAM(50),IPARAM(100)
      EQUIVALENCE (PARAM,IPARAM)
      EQUIVALENCE (N,IPARAM(1)),(M,IPARAM(2)),(A,IPARAM(3)) ,
1      (R,IPARAM(5)),(K,IPARAM(7)),(IFSTAN,IPARAM(8)),
2      (H,IPARAM(9)),(IFPTRANS,IPARAM(11)),(ISETS,IPARAM(12)),
3      (XFACT,IPARAM(13)),(GAIN,IPARAM(15))
      DIMENSION XX(1),YY(1)
      IF(KK.EQ.10) GO TO 940
      IF(KK)900,910,920
900  AK=K+4
      IXL=5
      IYL=8
      XT=1.2*AK
      YT=1.5
      XNG=-.1*AK
      YNG=-.4
      CALL AXISXY(4HTAPE,IXL,IYL,1.,XT,YT,XNG,YNG,.0,.0,.1)
      CALL PLOTXY(-.1*AK,-.4,0,0)
      GO TO 930
910  AM=M
      IXL=5
      IYL=8
      XT=1.2*AM
      YT=1.7
      XNG=-.1*AM
      YNG=-.6
      CALL AXISXY(4HTAPE,IXL,IYL,1.,XT,YT,XNG,YNG,.0,.0,.1)
      CALL PLOTXY(XNG,YNG,0,0)
      GO TO 930
920  AM=M
      IXL=5
      IYL=8
      XT=1.2*AM
      YT=1.2*XMAX
      YNG=-.15*XMAX
      XNG=-.1*AM
      CALL AXISXY(4HTAPE,IXL,IYL,1.,XT,YT,XNG,YNG,.0,.0,.01)
      CALL PLOTXY(XNG,YNG,0,0)
930  XX(1)=0.

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FORTRAN (3,2)/MASTER

09/10/71

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      JK=(II-1)/2
      YY(1)=ARRAYS(JK)
      CALL PLOTXY(XX,YY,0,15)
      CALL PLOTXY(XX,YY,1,15)
      JL=JJ+1
      DO 1000 I=1,JL
      LL=JK+I
      XX(1)=I
      YY(1)=ARRAYS(LL)
      CALL PLOTXY(XX,YY,1,15)
1000  CONTINUE
      RETURN
940  IXL=5
      IYL=40
      XT=1.2*(XMAX-XMIN)*FACT
      YT=1.1*JJ
      XNEG=1.2*XMIN*FACT
      YNEG=-.05*YT
      XTICK=1.
      CALL AXISXY(4HTAPE,IXL,IYL,XTICK,XT,YT,XNEG,YNEG,.0,.0,4.)
      CALL PLOTXY(XNEG,YNEG,0,0)
      YY(1)=1.
      JK=(II-1)/2
      XX(1)=ARRAYS(JK)*FACT
      CALL PLOTXY(XX,YY,0,15)
      CALL PLOTXY(XX,YY,1,15)
      JL=JJ-1
      DO 1100 I=1,JL
      LL=JK+I
      XX(1)=ARRAYS(LL)*FACT
      YY(1)=1.+I
      CALL PLOTXY(XX,YY,1,15)
1100  CONTINUE
      RETURN
      END

```

FORTRAN DIAGNOSTIC RESULTS FOR GRF

NO ERRORS

GRF P 00624 C 33260 D 00144

TABLE A-3.1

THE FUNCTION U_n , 4000 DATA POINTS

(The data is expressed in micrometers)

TABLE A-3.1 THE FUNCTION U_n , 4000 DATA POINTS (The data is expressed in micrometers)

SAMPLED DATA/ 3.4027E-01	5.4321E-01	5.0944E-01	5.3101E-01	8.4839E-01	6.2866E-01	4.7444E-01	3.6316E-01	2.6550E-01	1.4954E-01
-4.4270E-01	-3.2354E-01	-7.9349E-02	-2.9309E-01	-3.8157E-01	-4.7932E-01	-1.1296E-01	6.7749E-01	-1.6444E-01	1.4343E-01
1.1601E-01	1.7395E-01	3.5704E-01	3.1738E-01	1.9231E-01	2.9907E-01	9.3049E-01	-2.5339E-01	-4.3354E-01	-2.6865E-01
1.8005E-01	7.3242E-02	-1.7704E-01	1.0071E-01	1.8311E-02	2.9907E-01	4.7656E-02	-5.4955E-02	-4.0302E-01	1.3248E 00
-6.3648E-01	6.1035E-03	-2.4729E-01	-2.4118E-01	-2.8391E-01	-2.3203E-01	-2.1675E-01	-1.0381E-01	-1.8926E-01	-1.9226E-01
4.7607E-01	3.1124E-01	2.2848E-01	1.4038E-01	-3.5291E-02	5.9834E-01	-1.2822E-01	-2.2593E-01	-6.2885E-01	-1.0990E 00
-8.2130E-01	-5.7332E-01	-8.3941E-01	-4.6101E-01	-4.0607E-01	-2.5339E-01	3.2654E-01	-1.0681E-01	6.0120E-01	3.9673E-01
7.9346E-02	8.8501E-02	3.3264E-01	1.6249E-01	5.4955E-02	2.3699E-01	2.3699E-01	2.6550E-01	4.5776E-02	1.1597E-01
-6.5327E-01	-2.7475E-01	-5.8007E-02	-2.9297E-01	5.2814E-01	2.1342E-02	-1.8620E-01	-3.8462E-01	-3.1137E-01	-3.2358E-01
-4.0913E-01	-5.4035E-01	-2.6560E-01	-1.3738E-01	-3.5082E-01	-2.6235E-01	-2.8086E-01	2.9907E-01	-6.1059E-02	-2.3203E-01
-2.7475E-01	1.3073E-01	6.1625E-02	1.5259E-02	1.5265E-02	-1.4034E-01	-1.4034E-01	1.5259E-02	3.0529E-02	3.9978E-01
-5.7007E-02	7.3242E-02	-1.5265E-02	-1.0686E-01	1.2512E-01	-9.7704E-02	2.7466E-02	2.3499E-01	1.5869E-01	1.2207E-01
3.673E-02	3.3264E-01	5.7087E-01	-3.1137E-01	-2.4426E-02	-6.1059E-02	4.2725E-02	1.4954E-01	1.8616E-01	4.4250E-01
3.1733E-01	2.2593E-01	2.2848E-01	4.7913E-01	4.3640E-01	4.1804E-01	3.2654E-01	5.4932E-02	9.1552E-03	3.9673E-01
-1.5266E-01	1.3123E-01	1.3123E-01	1.4038E-01	3.9266E-01	5.3406E-01	5.6152E-01	3.1433E-01	3.4790E-01	-7.3266E-02
-4.7627E-01	-2.7781E-01	-2.3813E-01	-3.6631E-01	-1.8315E-01	-1.4348E-01	-5.0378E-02	-1.6179E-01	-1.0076E-01	-9.7704E-02
-1.6789E-01	-2.2439E-01	-3.0529E-02	3.3264E-01	-5.4955E-02	1.2817E-01	5.1880E-02	1.5544E-01	6.1035E-01	-9.1577E-02
-9.3422E-01	-7.1125E-01	-5.8003E-01	-5.4035E-01	-2.6865E-01	-5.5256E-01	-4.8237E-01	-3.3274E-01	1.5259E-02	-1.7705E-01
1.1902E-01	1.7395E-01	-2.7475E-02	6.1035E-02	6.7139E-02	-3.4494E-01	-4.7781E-01	-2.3203E-01	-1.1296E-01	-1.9446E-01
6.4047E-02	-1.4958E-01	9.1577E-02	1.7395E-01	1.2207E-01	4.7302E-02	9.6045E-02	4.2725E-01	9.8305E-01	-3.2969E-01
-1.1296E-01	-2.9011E-01	3.5715E-01	-8.8525E-02	-6.4411E-01	-5.732E-01	-9.6005E-01	-8.6406E-01	7.4768E-01	1.5869E-01
-1.3523E 00	-2.0126E 00	-1.0164E 00	-6.8378E-01	-9.2505E-01	-5.4035E-01	1.0375E-01	-6.7162E-02	3.3569E-02	-1.0990E 00
7.3242E-02	-2.0151E-01	-8.2421E-02	-8.5473E-02	2.0375E-01	3.0517E-03	4.1035E-03	5.4932E-02	2.1362E-01	-1.5265E-02
1.4343E-01	2.7161E-01	2.5635E-01	2.9907E-01	1.0375E-01	7.6249E-02	6.7139E-02	4.6692E-01	1.0651E 00	3.1738E-01
3.6621E-01	2.2583E-01	8.2397E-02	1.2911E-01	-1.0381E-01	2.7466E-02	1.4343E-01	1.8005E-01	2.9602E-01	2.831E-01
1.5869E-01	1.3733E-01	2.0142E-01	2.7466E-02	-2.3508E-01	8.2307E-02	-2.3508E-01	8.2307E-02	1.0691E-01	-3.5410E-01
9.7704E-01	-6.4441E-01	-5.5256E-01	-6.3801E-01	-4.2744E-01	5.6171E-01	-3.1748E-01	-2.3813E-01	-5.1904E-02	2.7466E-02
6.7444E-01	6.7444E-01	1.4646E-01	2.5496E-01	3.5566E-01	3.7231E-01	9.1553E-02	-1.9446E-01	-1.0381E-01	-3.432E-01
-3.9692E-01	3.9692E-01	-1.7400E-01	1.6789E-01	1.8010E-01	3.7231E-01	3.1510E-02	-4.5795E-01	-1.6484E-01	-9.458E-01
-9.4938E-01	9.525E-01	-1.1448E 00	-1.2302E 00	-9.6779E-01	-5.9528E-01	-6.1054E-01	6.7139E-02	-2.8255E-01	-1.1601E-01
-3.8767E-01	1.785E-01	-1.9314E 00	-1.2312E-01	-2.1067E-01	-1.5549E-01	-1.2212E-01	-3.0272E-01	-4.5788E-02	-7.0820E-01
-3.0529E-02	1.3428E-01	3.0222E-01	3.8767E-01	-3.3581E-02	2.7161E-01	2.3499E-01	6.4077E-02	3.5400E-01	2.3804E-01
1.8616E-01	-1.7705E-01	-3.9645E-02	-1.5569E-01	-8.2421E-02	-3.3581E-02	-1.7705E-01	-8.8525E-02	2.7161E-01	1.6479E-01
-5.4955E-02	4.2725E-02	-3.9645E-02	3.0529E-02	1.8311E-01	4.2725E-02	8.2397E-02	-1.9846E-01	3.9673E-02	-1.2822E-01
3.6521E-02	-7.9369E-02	4.5776E-02	5.1880E-02	-1.5265E-02	4.2736E-02	-4.6406E-01	-1.9846E-01	-4.4880E-01	-6.4716E-01
-2.0151E-01	9.7456E-02	2.3499E-01	2.2583E-01	3.4790E-01	2.1973E-01	1.7395E-01	2.5024E-01	1.4343E-01	4.3640E-01
-3.9692E-01	-1.1907E-01	-9.4028E-02	-9.4028E-02	1.0681E-01	1.2512E-01	2.4715E-01	2.3499E-01	-5.1904E-02	-2.7170E-01
-1.0991E-01	-2.1372E-01	-2.0151E-01	-2.5560E-01	-7.9349E-02	3.0518E-02	1.7395E-01	7.0190E-02	5.0659E-01	7.0496E-01
8.3923E-01	7.3853E-01	6.6223E-01	2.7771E-01	1.8005E-01	-6.7162E-02	-1.9846E-01	-2.4426E-02	9.1553E-02	3.9673E-01
2.4109E-01	1.2264E 00	3.7842E-01	4.5387E-01	4.1809E-01	9.1553E-02	4.3640E-01	1.0986E-01	2.3499E-01	4.0494E-01
4.6082E-01	3.5095E-01	2.5635E-01	2.2278E-01	-1.3127E-01	-6.5327E-01	5.4340E-01	7.3090E-01	2.6855E-01	-1.1296E-01
-3.7241E-01	-3.8462E-01	-3.6631E-01	-4.4575E-01	-2.8636E-01	-5.6171E-01	-2.1368E-02	1.1891E-01	6.5613E-01	5.4932E-01
1.8311E-01	1.3123E-01	-7.3264E-02	-5.9007E-02	1.1526E-03	4.2725E-02	3.9063E-01	2.0752E-01	3.9063E-01	2.1362E-01
4.4861E-01	3.7842E-01	4.637E-01	1.9479E-01	1.8005E-01	2.1362E-01	1.0071E-01	1.1296E-01	2.7466E-02	-3.7546E-01
-3.4800E-01	-3.0222E-01	-1.0441E 00	-8.4003E-01	1.9226E-01	-7.3266E-02	1.0681E-01	-2.4426E-02	8.8501E-02	-2.2827E-01
-1.0070E-01	1.6479E-01	1.9479E-01	6.1035E-01	6.1035E-01	-1.3127E-01	-2.7475E-01	-2.4426E-02	-2.8696E-01	-1.8926E-01
-1.3738E-01	-2.5034E-01	-8.8525E-02	-5.3425E-01	-4.5490E-01	-5.3120E-01	-4.4575E-01	-6.9294E-01	-5.1899E-01	-3.0222E-01
8.2397E-02	4.2419E-01	-4.5195E-01	-2.729E-01	-1.7705E-01	-4.4575E-01	-3.3579E-01	-4.2133E-01	-5.7697E-01	-2.5339E-01
-9.7704E-02	2.5944E-01	5.4932E-02	6.087E-02	-1.2822E-01	-1.9846E-01	-1.3274E-01	-1.043E-01	-2.1368E-02	-9.8915E-01
-4.6101E-01	6.1035E-03	-2.4426E-02	-8.1286E-01	-8.0299E-01	-1.2363E 00	-1.3615E 00	-1.2821E 00	-1.1082E 00	-1.1265E 00
-1.0990E 00	-1.1533E 00	-1.4042E 00	-9.1286E-01	-3.4494E-01	-3.748E-01	-8.8918E-01	-6.5937E-01	-1.3147E 00	-1.1112E 00
-6.4106E-01	-1.3738E-01	-6.9904E-01	-2.9306E-01	-2.2643E-01	-3.7546E-01	-1.0076E-01	-4.5185E-01	-3.4494E-01	1.3428E-01
1.5869E-01	4.4550E-01	8.8501E-02	-5.6171E-01	-6.9294E-01	-5.3120E-01	-4.4575E-01	-1.4958E-01	-1.8620E-01	-2.9612E-01
-3.0529E-01	-3.9692E-01	-3.9692E-01	-4.4270E-01	-3.4494E-01	-1.8315E-01	-1.0686E-01	-1.2213E-02	3.9673E-02	-2.1368E-02

1.7090E-01	-1.3734E-01	-1.5264E-01	-4.5788E-02	8.2397E-02	-1.3127E-01	-2.9306E-01	-8.2436E-01	-1.0960E 00	-5.6171E-01
-9.3422E-01	-7.6614E-01	-5.2814E-01	-3.4494E-01	7.6229F-02	4.2770E-02	-1.8620F-01	-3.0832E-01	-3.8462E-01	-3.7546E-01
-1.8620E-01	-5.6633E-02	-4.9743E-01	-4.2439E-01	1.2220F-01	1.7700E-01	-3.3123F-01	-8.2421E-02	-2.5949E-01	1.8311E-01
6.3782E-01	5.2490E-01	2.2888E-01	3.9063E-01	3.9063E-01	2.5335E-01	-2.3182F-01	2.2728E-01	2.9907E-01	6.4087E-02
-3.9048E-01	-6.0139E-01	-2.9304E-01	-1.9846E-01	-2.2247E-01	-2.1368E-02	-9.1582E-03	8.8501E-02	6.7139E-02	3.4485E-01
2.0752E-01	2.9907E-01	7.6224E-02	3.5706E-01	2.6245F-01	1.1597E-01	4.0425E-01	3.2654E-01	-4.1523E-01	-2.6560E-01
-1.4938E-01	6.4087E-02	-2.1677E-02	-4.5788E-02	4.2725F-02	-4.5788E-02	-2.4426E-02	1.0841E-01	-2.6885E-01	7.0190E-02
6.6223E-01	2.3193E-01	4.2725E-02	-6.4111E-02	1.8926E-01	1.5569E-01	-7.0214E-02	-1.3035E 00	-1.3941E 00	-1.2363E 00
-7.1430E-01	-9.0605E-01	-7.2346E-01	4.2725E-02	-5.1944E-02	1.8010E-01	-2.0456F-01	-3.7241E-01	-1.5561E-01	-5.6782E-01
-6.2275E-01	-9.8610E-01	-6.2885E-01	-6.3801E-01	7.2651E-01	-5.8918E-01	3.5706F-01	4.3335E-01	2.5635E-01	3.1128E-01
3.9673E-01	2.7161E-01	8.2397E-02	1.3123E-01	8.549E-02	-2.0761E-01	-4.4111E-02	2.6855E-01	1.0986E-01	3.0518E-02
3.5400E-01	1.7700E-01	4.4556E-01	2.8922E-01	5.4626F-01	-2.8391E-01	-1.4938E-01	1.6484E-01	1.577E-02	1.5259E-01
-2.8456E-01	-9.7694E-01	-1.4404E 00	6.7444E-01	9.9487E-01	3.4790E-01	5.0049E-01	5.7983E-02	-2.1067E-01	-3.3581E-02
4.2144E-01	1.6784E-01	9.7656E-02	1.1597E-01	9.7704E-02	1.3738E-01	8.549E-02	9.1552E-03	6.7162E-01	2.7161E-01
2.2183E-01	6.3782E-01	4.8824E-01	3.1728E-01	2.8922E-01	3.6011E-01	3.3875F-01	3.1738E-01	1.9228E-01	2.2888E-01
1.2207E-01	4.1809E-01	1.0420E 00	7.7820E-01	1.1149E 00	5.0289E-01	4.4861F-01	7.6599E-01	2.0142E-01	2.8381E-01
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-3.1748E-01	-5.4038E-01	-2.5949E-01	2.1667E-01	6.4697E-01	1.9531E-01	2.4719E-01	5.8932E-02	-2.9306E-01	-4.5490E-01
-9.7704E-02	-1.9444E-01	-6.4880E-01	1.9231E-01	-2.3813F-01	-7.2041E-01	-3.5410F-01	-5.8918E-01	-2.4576E-01	2.7161E-01
2.5330E-01	1.4034E-01	-3.0529E-02	2.0752E-01	-6.4106F-01	-4.4575E-01	-5.6171F-01	-9.4458E-01	-2.5949E-01	-4.2736E-02
9.7656E-02	2.5330E-01	-3.2663E-01	1.9531E-01	3.0212F-01	7.0190E-02	4.8501E-02	5.1880E-02	-9.7704E-02	2.8687E-01
1.3123E-01	8.9414E-01	2.4414E-01	-1.2817E-01	-1.2517E-01	1.1601E-01	-2.4729E-01	7.0214E-02	-1.8316E-02	2.5330E-01
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-3.9082E-01	-3.9082E-01	-3.9082E-01	0	-3.9082E-01	-3.9082E-01	-3.7851E-01	-3.7851E-01
-3.6325E-01	-3.5105E-01	-3.5105E-01	-3.4800E-01	-3.4800E-01	-3.4800E-01	-3.4800E-01	-3.4800E-01
-3.3579E-01	-3.3579E-01	-3.3579E-01	-3.3274E-01	-3.3274E-01	-3.3274E-01	-3.3274E-01	-3.3274E-01
-3.1443E-01	-3.1443E-01	-3.1443E-01	-2.9612E-01	-2.9612E-01	-2.9612E-01	-3.1443E-01	-3.1443E-01

NUMBER= 3978

AVERAGE=-4.46981E-02

STANDARD DEVIATION= 4.30792E-01

MINIMUM=-2.27737E 00

MAXIMUM= 1.64185E 00

TABLE A-3.2

THE DISCRETE FUNCTION z_n , 4000 DATA POINTS

(The data is in nondimensional form for conversion to micrometers, a factor σ_y should be utilized)

TABLE A-3.2 THE DISCRETE FUNCTION Z_n , 4000 DATA POINTS (The data is in nondimensional form for conversion to micro-meters, a factor σ should be utilized^y)

THE DATA HAVE BEEN NORMALISED AND STANDARDISED

MINIMUM=-5.18272E 00
 MAXIMUM= 3.91498E 00
 STANDARD DEVIATION=1.0

8.9363E-01	1.3647E 00	1.2848E 00	1.3364E 00	2.0731E 00	1.5631E 00	1.6693E 00	9.4676E-01	7.2007E-01	4.5088E-01
-9.2387E-01	-6.4737E-01	-8.0483E-02	-5.7553E-01	-7.6191E-01	-1.0089E 00	-1.5848E-01	1.6764E 00	-2.7889E-01	4.3671E-01
1.6555E-01	9.0755E-01	9.3259E-01	8.4505E-01	3.4626E-01	7.9277E-01	-8.9259E-01	-4.8444E-01	-9.0262E-01	-5.1986E-01
5.2172E-01	3.0727E-01	3.0727E-01	3.0727E-01	1.4626E-01	7.9900E-01	3.3045E-01	-2.3811E-02	-8.3178E-01	-2.9716E 00
-1.13737E 00	1.1793E-01	-4.7027E-01	-4.5610E-01	-5.5528E-01	-4.3485E-01	-3.8526E-01	-1.3721E-01	-3.3556E-01	5.5005E-01
1.2089E 00	8.2633E-01	6.3506E-01	4.2662E-01	3.2889E-02	-1.2852E 00	-1.9388E-01	-2.8688E-01	-1.3560E 00	-2.4474E 00
-1.8027E 00	-1.2285E 00	-1.8452E 00	-9.6638E-01	3.8889E-01	-4.8444E-01	8.6175E-01	3.5170E-01	1.4993E 00	1.0247E 00
2.7794E-01	3.0920E-01	7.5992E-01	4.8630E-01	-2.3811E-02	3.2890E-02	4.923E-01	7.2007E-01	3.7295E-01	3.7295E-01
-1.1127E 00	-4.7027E-01	3.0895E-02	7.8383E-01	-1.1222E 00	1.5335E-01	-3.2848E-01	7.8906E-01	-6.1904E-01	-6.4737E-01
-8.4595E-01	-1.1504E 00	-5.1278E-01	-2.1514E-01	-8.0344E-01	-5.0569E-01	-5.4820E-01	7.9800E-01	-3.7979E-02	-4.3485E-01
-5.3403E-01	-2.1514E-01	-5.2147E-02	1.3918E-01	6.8324E-02	-2.2222E-01	-2.2222E-01	1.3918E-01	3.2890E-02	1.0318E 00
-3.0895E-02	2.7378E-01	6.8324E-02	1.4430E-01	3.9420E-01	-1.2304E-01	1.6751E-01	6.4923E-01	4.7213E-01	3.8712E-01
1.9595E-01	5.1278E-01	-1.2214E 00	-6.1904E-01	4.7059E-02	-3.7979E-02	2.0293E-01	4.5088E-01	5.3589E-01	1.1309E 00
8.7592E-01	6.2798E-01	6.3506E-01	1.2160E 00	1.1168E 00	1.0743E 00	8.6175E-01	2.3127E-01	1.2501E-01	1.0247E 00
8.4050E-01	4.6504E-01	4.0837E-01	4.2662E-01	9.6693E-01	1.3435E 00	1.4072E 00	8.3342E-01	9.1134E-01	-6.6315E-02
-2.5036E-01	-6.1904E-01	-1.0301E 00	-6.1195E-01	-3.7145E-01	-2.2930E-01	-1.3185E-02	-2.7181E-01	1.3013E-01	-1.2304E-01
-1.0018E 00	-5.4111E-01	-4.4902E-01	-6.5446E-01	-7.4655E-01	-3.6401E-01	4.0837E-01	4.6504E-01	2.4544E-01	-1.0882E-01
-2.8598E-01	-8.8137E-01	3.2890E-02	8.7592E-01	-2.3811E-02	4.0129E-01	2.2419E-01	4.6504E-01	1.3918E-01	-3.0723E-01
-2.8048E 00	-1.5473E 00	-1.2427E 00	-1.1506E 00	-5.1986E-01	-1.1789E 00	-1.0160E 00	-6.863E-01	-1.5848E-01	-3.5693E-01
3.6064E-01	5.0755E-01	3.9974E-02	2.4544E-01	2.5961E-01	-6.9696E-01	-5.4111E-01	-4.3485E-01	-2.1782E-01	-6.6154E-01
5.7839E-01	-2.347E-01	-1.0882E-01	5.0755E-01	3.8712E-01	2.1356E-01	3.2336E-01	1.0955E 00	1.8394E 00	4.7213E-01
2.5252E-01	-2.347E-01	-6.7571E-01	-7.2530E-01	-1.3919E 00	-1.2885E 00	-1.1767E 00	-9.7346E-01	-2.0097E-01	-5.4111E-01
-5.845E-01	-5.945E-01	-7.2530E-01	-1.0174E-01	1.9388E-01	-1.7972E-01	1.0882E-01	3.7440E-01	1.8168E-01	-2.4474E 00
-2.0334E 00	-2.9433E 00	-2.2561E 00	-1.4835E 00	-2.0436E 00	-1.1506E 00	3.4462E-01	-5.2147E-02	1.7263E-01	1.8168E-01
2.7378E-01	3.6401E-01	-8.7567E-02	-9.4651E-02	-2.0436E 00	1.1084E-01	1.1793E-01	2.3127E-01	5.9964E-01	6.8324E-02
4.3671E-01	7.3424E-01	6.982E-01	7.9800E-01	3.4462E-01	2.8086E-01	2.5961E-01	1.8766E 00	2.5761E 00	8.4050E-01
9.5385E-01	6.2798E-01	2.9504E-01	3.6587E-01	1.3721E-01	1.6751E-01	4.3671E-01	5.2172E-01	7.9091E-01	7.6258E-01
4.7213E-01	4.2254E-01	5.7131E-01	3.6871E-01	5.4150E-02	1.6751E-01	-4.4194E-01	2.9504E-01	3.5170E-01	-7.1821E-01
-1.4693E 00	-1.3914E 00	-1.1789E 00	-1.3773E 00	-8.845E-01	-1.2002E 00	-4.3321E-01	-4.4902E-01	-1.6727E-02	1.6751E-01
3.2336E-01	1.6693E 00	4.4379E-01	-4.9861E-01	8.8300E-01	9.6801E-01	3.1628E-01	-3.5693E-01	-1.3721E-01	-2.0605E-01
-8.8444E-01	-8.1761E-01	-3.0014E-01	-2.8598E-01	3.1431E-01	1.1637E-02	-7.1113E-01	-9.5929E-01	-2.7889E-01	-1.0443E 00
-2.1003E 00	-2.1074E 00	-2.5536E 00	-2.7520E 00	-2.1429E 00	-2.781E 00	-1.3135E 00	2.5961E-01	-5.0569E-01	-1.6555E-01
-7.9614E-01	4.9338E-01	6.1240E-02	-1.7972E-01	3.8526E-01	-2.5764E-01	-1.7972E-01	-5.9779E-03	-2.5307E-03	-1.5402E 00
-4.5610E-01	3.2890E-02	4.1544E-01	-1.1590E-01	2.5805E-02	7.3424E-01	6.4923E-01	2.5252E-01	9.2551E-01	6.5631E-01
-5.3589E-01	-3.0723E-01	-5.9779E-01	-7.9014E-01	-4.9861E-01	-4.0522E-01	5.0047E-01	-1.0174E-01	7.3424E-01	4.8630E-01
-2.3811E-02	2.0293E-01	1.1637E-02	-2.5764E-01	8.7567E-02	2.5805E-02	-1.0723E-01	7.6966E-01	1.9585E-01	-1.9388E-01
1.8877E-01	-8.0483E-02	1.1637E-02	3.2890E-02	5.2889E-01	2.0293E-01	2.9504E-01	-3.5693E-01	-9.3804E-01	-1.3985E 00
-1.4268E 00	-7.8197E-01	2.1002E-01	2.2419E-01	6.8324E-02	4.5344E-03	-9.7346E-01	-3.4265E-01	5.7131E-01	2.1710E-01
-3.6401E-01	3.045E-01	6.4923E-01	6.2798E-01	9.1134E-01	6.1381E-01	5.0755E-01	6.8465E-01	4.3671E-01	1.1168E 00
1.2018E 00	1.1637E-02	-1.7263E-01	-1.1590E-01	3.5170E-01	3.9420E-01	4.7757E-01	6.4923E-01	-1.6727E-02	-5.2694E-01
-1.5138E-01	-3.0235E-01	-3.6401E-01	-5.1278E-01	-8.0483E-02	1.7460E-01	5.0755E-01	2.6669E-01	1.2797E 00	1.7402E 00
2.0519E 00	1.8181E 00	1.6410E 00	7.4841E-01	5.2172E-01	-5.2147E-02	-3.5693E-01	4.7058E-02	3.1628E-01	1.0247E 00
6.6340E-01	2.9515E 00	9.8218E-01	1.1805E 00	1.6743E 00	3.1628E-01	1.1168E 00	3.5894E-01	6.4923E-01	1.0530E 00
1.1735E 00	9.1842E-01	6.9882E-01	6.2089E-01	-2.0097E-01	-1.4127E 00	-1.1576E 00	1.8004E 00	7.2716E-01	-1.5846E-01
-6.6072E-01	-7.8906E-01	-7.4655E-01	-9.3096E-01	5.6233E-01	-1.2002E 00	5.4150E-02	3.6587E-01	1.6268E 00	1.3789E 00
-5.2889E-01	4.0837E-01	-6.6315E-02	3.0895E-02	1.2501E-01	2.0293E-01	1.0105E 00	5.8547E-01	1.0105E 00	5.9964E-01
1.1431E 00	9.8218E-01	1.1805E 00	4.8630E-01	3.5170E-01	2.9649E-01	3.7375E-01	-1.5846E-01	1.6751E-01	-7.6780E-01
-7.0405E-01	-5.9779E-01	-2.3199E 00	1.9019E 00	5.4054E-01	-6.6315E-02	3.5170E-01	6.6694E-01	3.0920E-01	-4.1360E-01
-1.3013E-01	4.8630E-01	5.6422E-01	-2.4347E-01	-2.0097E-01	-2.0097E-01	-5.3403E-01	4.7058E-02	5.6237E-01	-3.3556E-01
-2.1514E-01	-4.7736E-01	-1.0174E-01	-1.1364E 00	-9.5221E-01	-1.1293E 00	-9.3804E-01	-1.5848E 00	-1.1010E 00	-5.9779E-01
2.9503E-01	1.0884E 00	-9.4513E-01	-4.7027E-01	-3.0723E-01	-9.3096E-01	-4.7027E-01	-8.7426E-01	1.2356E-01	-8.4444E-01
-1.2304E-01	7.0590E-01	2.5252E-01	2.5252E-01	-1.9388E-01	3.3556E-01	-6.663E-01	-2.2222E-01	5.4150E-02	-2.1924E 00
-9.6638E-01	1.1793E-01	4.7054E-02	-5.1986E-01	-1.7602E 00	-2.7662E 00	-3.0566E 00	-2.8724E 00	-2.4686E 00	-2.5111E 00

-2.4474E 00	-2.5749E 00	-3.1558E 00	-2.0153E 00	-6.9699E-01	-6.3321E-01	-1.2639E 00	-1.4268E 00	-2.9574E 00	-2.4757E 00
-1.3843E-01	-2.1514E-01	-1.5149E 00	-5.7653E-01	-6.5446E-01	-7.6780E-01	-1.3013E-01	-9.4513E-01	-6.9696E-01	-4.1546E-01
-4.7213E-01	-1.380E-01	-3.0920E-01	-1.202E 00	-1.5048E 00	-1.1293E 00	-9.3096E-01	-2.4347E-01	-3.2848E-01	-5.8362E-01
-6.0487E-01	-8.1761E-01	-8.1761E-01	-9.2387E-01	-6.9699E-01	-3.2140E-01	-1.4430E-01	7.5408E-02	1.9585E-01	-4.1566E-02
-5.0047E-01	-2.1514E-01	-2.5056E-01	-2.5056E-01	-2.9503E-03	-2.0079E-01	-5.7653E-01	-1.8098E 00	-2.4403E 00	-1.2002E 00
-2.0648E 00	-1.6748E 00	-1.1227E 00	-6.9496E-01	2.8088E-01	2.0293E-01	2.0293E-01	-6.1195E-01	-7.8906E-01	-5.2880E-01
-3.2848E-01	-1.8721E-02	-1.0514E 00	-8.6137E-01	-4.8717E-01	5.1643E-01	4.0837E-01	-8.7567E-02	-4.9461E-01	2.5525E-01
-1.5844E 00	1.6410E 00	1.3222E 00	6.3066E-01	1.0103E 00	6.9842E-01	4.4902E-01	8.2089E-01	7.9800E-01	9.0426E-01
-8.0344E-01	-1.2022E 00	-5.7653E-01	-3.5693E-01	-4.1360E-02	5.4156E-02	8.2499E-02	3.0920E-01	2.5961E-01	9.0426E-01
-5.8547E-01	7.9800E-01	2.8088E-01	3.3595E-01	7.1299E-01	3.7295E-01	4.7058E-02	8.6175E-01	-8.6012E-01	-5.1278E-01
-2.4347E-01	2.5525E-01	-3.9943E-01	6.2089E-01	2.0293E-01	-2.5307E-03	4.7058E-02	3.5170E-01	-5.1986E-01	2.6669E-01
1.6410E 00	6.4215E-01	2.0293E-01	-4.5063E-02	-3.3559E-01	-2.5307E-03	-5.9231E-02	-2.9220E 00	-3.1416E 00	-2.7662E 00
-1.5544E 00	1.673E 00	-1.5756E 00	2.0293E-01	-1.6727E-02	-3.1710E-01	-3.7110E-01	-6.7072E-01	-2.7644E-01	-1.2143E 00
-1.3418E 00	-2.153E 00	-1.3560E 00	-1.3773E 00	-1.5827E 00	-1.2639E 00	9.3259E-01	1.1079E 00	6.9882E-01	8.2633E-01
1.0247E 00	7.3424E-01	2.9503E-01	4.0837E-01	3.0211E-01	-3.7818E-01	-4.5063E-02	7.2716E-01	3.5878E-01	1.7460E-01
9.2551E-01	5.1463E-01	1.1380E 00	7.674E-01	1.3718E 00	-5.5528E-01	-2.4347E-01	-2.7889E-01	-1.0882E-01	4.5796E-01
3.4462E-01	9.2551E-01	1.7543E 00	1.6693E 00	2.4132E 00	9.1134E-01	1.2655E 00	2.3435E-01	-3.8526E-01	2.5805E-02
-5.6237E-01	-2.1640E 00	-3.2408E 00	-2.4261E 00	-1.7177E 00	-2.1514E-01	3.0211E-01	1.2501E-01	-5.2147E-02	7.3424E-01
1.0814E 00	4.9338E-01	3.3045E-01	3.7295E-01	-1.2304E-01	4.6504E-01	8.909E-01	8.4050E-01	5.5005E-01	6.3506E-01
6.2798E-01	1.5947E 00	1.2372E 00	8.4050E-01	7.674E-01	9.3988E-01	1.1451E 00	1.8819E 00	5.7131E-01	7.6258E-01
1.4993E 00	9.5385E-01	1.2443E 00	6.3040E-01	2.1002E-01	4.7058E-02	7.6258E-01	8.6175E-01	1.5206E 00	1.3435E 00
-6.3321E-01	-4.2962E-01	-4.9861E-01	6.0673E-01	1.6056E 00	5.5714E-01	-7.8526E-01	-5.1986E-01	-5.7653E-01	-9.6181E-01
-1.2304E-01	-3.5693E-01	-9.3804E-01	-3.4265E-01	-4.4902E-01	-1.5685E 00	-6.7757E-01	-1.2639E 00	-4.6673E-01	7.3424E-01
6.9173E-01	4.2962E-01	3.2890E-02	5.8547E-01	-1.3843E 00	-9.3096E-01	1.2002E 00	-1.0443E 00	4.0129E-01	7.6966E-01
3.3045E-01	6.9173E-01	6.5448E-01	5.5714E-01	2.2219E 00	1.1309E 00	1.3222E 00	-1.1590E-01	4.0129E-01	3.3753E-01
4.0837E-01	2.1794E 00	6.7048E-01	4.0129E-01	-1.8680E-01	-1.6555E-01	-4.7027E-01	-2.2419E-01	-1.2304E-01	6.9173E-01
2.7378E-01	-2.2222E-01	-1.3914E 00	-1.0797E 00	8.508E-01	2.6669E-01	3.920E-01	-5.9231E-02	6.1240E-02	6.9173E-01
2.5252E-01	-9.3096E-01	-1.0939E 00	-6.3321E-01	9.1842E-01	7.6966E-01	8.2499E-02	7.2716E-01	4.9338E-01	6.6340E-01
8.3345E-01	6.340E-01	4.7058E-02	-3.1431E-01	9.9635E-01	6.2798E-01	-9.4651E-02	5.4297E-01	4.9338E-01	1.1097E 00
4.0129E-01	-8.7567E-02	-2.4347E-01	-8.7567E-02	2.6419E-01	-5.9231E-02	1.8721E-02	-8.4595E-01	-5.5528E-01	2.8088E-01
1.3435E 00	9.5385E-01	9.5385E-01	3.3045E-01	-2.9306E-01	-4.1111E-01	3.9844E-01	1.2285E 00	-2.7181E-01	-7.8197E-01
1.0374E-01	5.3599E-01	4.1544E-01	3.3045E-01	7.3424E-01	8.0508E-01	2.8794E-01	5.0047E-01	1.1637E-02	-1.3721E-01
1.3364E 00	5.9944E-01	1.1644E 00	8.909E-01	4.6504E-01	3.1628E-01	4.8324E-02	-6.6154E-01	-1.4693E 00	-1.8665E 00
3.6587E-01	-8.7567E-01	-3.2848E-01	2.0293E-01	2.2419E-01	3.9974E-02	4.8324E-02	2.3835E-01	6.9173E-01	8.8300E-01
1.1168E 00	1.4781E 00	3.0920E-01	4.6504E-01	3.1628E-01	-2.5307E-03	5.4156E-02	1.5737E 00	1.8110E 00	1.8110E 00
1.2797E 00	1.537E 00	9.3259E-01	5.2172E-01	-6.9696E-01	-1.4693E 00	-2.8441E 00	-2.7024E 00	-6.1195E-01	3.9974E-02
3.3045E-01	1.3010E 00	5.8297E-01	4.3671E-01	3.5170E-01	3.0211E-01	7.8004E-01	-4.7027E-01	-1.9388E-01	-2.2930E-01
8.6175E-01	1.4072E 00	1.3435E 00	1.3435E 00	7.674E-01	5.5714E-01	-1.2304E-01	4.0837E-01	-5.0569E-01	-2.2222E-01
-1.0301E 00	-1.0089E-01	-9.7344E-01	-6.9696E-01	-1.0174E-01	-5.1986E-01	-1.5402E 00	1.1451E 00	-1.0160E 00	-6.6508E-01
8.3345E-01	-1.7972E-01	3.7295E-01	4.5088E-01	4.8630E-01	8.1216E-01	1.0672E 00	1.3489E 00	2.2998E 00	1.9739E 00
1.1947E 00	5.6422E-01	4.0129E-01	9.8927E-01	4.2254E-01	6.5631E-01	9.1216E-01	1.1876E 00	1.0105E 00	4.0837E-01
1.9810E 00	1.2372E 00	6.2089E-01	9.6801E-01	9.1842E-01	6.6340E-01	7.3424E-01	1.6127E 00	1.9527E 00	2.0448E 00
8.9009E-01	1.5702E 00	7.1299E-01	9.1842E-01	4.7921E-01	1.5335E-01	2.1002E-01	5.0047E-01	-4.0652E-01	-2.0805E-01
-3.2848E-01	4.6504E-01	1.0374E-01	4.6504E-01	7.2716E-01	-3.4265E-01	1.3151E 00	1.0743E 00	1.5277E 00	1.5277E 00
-2.5891E 00	5.9256E-01	3.1853E 00	3.0211E-01	-1.0868E 00	-1.3135E 00	1.3914E 00	-1.2143E 00	-1.0656E 00	-1.5119E 00
2.9919E 00	1.8604E 00	2.5252E-01	8.909E-01	9.7510E-01	1.1593E 00	1.3435E 00	-3.7979E-02	5.7839E-01	1.6751E-01
5.0047E-01	1.8181E 00	1.2939E 00	1.0884E-01	1.4851E 00	2.4061E 00	1.7756E 00	2.160E 00	4.0129E-01	1.7323E-01
-1.8948E-01	-2.9645E 00	-2.1003E 00	-1.0726E 00	-1.0443E 00	-2.2930E-01	4.254E-01	-2.3811E-02	-3.8526E-01	-1.0656E 00
-2.5041E 00	-2.1337E 00	-1.1647E 00	3.5170E-01	5.5714E-01	-2.4347E-01	3.9420E-01	7.3424E-01	5.4297E-01	2.5252E-01
4.0837E-01	5.1463E-01	2.7181E-01	1.2514E 00	-2.7889E-01	7.3947E-01	3.2336E-01	1.8877E-01	2.5805E-02	1.6751E-01
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5.2127E-01	1.4922E 00	1.4922E 00	-7.3399E-02	-7.6235E 00	7.6235E 00	2.2419E-01	-1.4430E-01	1.5335E-01	4.2962E-01
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-3.9235E-01	-2.0578E 00	-2.0578E 00	-2.2065E 00	-2.3695E 00	-1.5544E 00	-1.7815E 00	-1.7532E 00	-1.6252E 00	-2.0861E 00
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4.0837E-01	4.3671E-01	4.3671E-01	2.9503E-01	-1.6737E-02	5.9505E-01	9.3968E-01	2.1002E-01	2.2419E-01	1.1876E 00
4.6884E-01	1.0034E 00	1.1593E 00	3.8712E-01	2.0293E-01	1.9388E-01	-2.5693E-01	1.0376E-01	7.2530E-01	4.9861E-01
5.3599E-01	1.0247E 00	1.0247E 00	1.2522E 00	1.3647E-01	3.5170E-01	5.9944E-01	2.1002E-01	1.3918E-01	-3.0723E-01
-3.3556E-01	1.3718E 00	1.3718E 00	1.2655E 00	7.9800E-01	4.2554E-01	7.1299E-01	1.8884E-01	7.2007E-01	8.5467E-01
7.6258E-01	6.6340E-01	6.6340E-01	-1.2304E-01	1.4072E 00	-6.6315E-02	-1.9161E-01	1.8721E-02	1.8721E-02	6.6340E-01
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4.7058E-02	4.0255E-01	6.8465E-01	1.9178E-01	-4.9402E-01	5.2172E-01	5.2172E-01	-3.5693E-01	-2.5056E-01	5.4297E-01
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1.0459E 00	1.4354E 00	1.3435E 00	1.6127E 00	1.2939E 00	9.8218E-01	1.4354E 00	3.8712E-01	1.4851E 00	8.9009E-01
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3.8712E-01	3.5251E 00	8.5447E-01	4.1546E-01	1.5631E 00	1.2018E 00	1.381E-01	9.8927E-01	8.6175E-01	9.8927E-01
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7.863E-01	1.8877E-01	5.9070E-01	1.5846E-01	-7.567E-02	6.2049E-01	2.1002E-01	4.7213E-01	2.5805E-02	-3.9235E-01
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6.6340E-01	8.0504E-01	4.0837E-01	3.1628E-01	6.4923E-01	4.0129E-01	1.1380E-00	1.4426E-00	6.2798E-01	3.3837E-01
1.3505E-01	2.4544E-01	4.0129E-01	-3.8526E-01	5.0755E-01	7.2716E-01	-1.5138E-01	-0.652E-01	-8.0483E-02	-1.6555E-00
-1.0658E-00	-1.2710E-00	-1.3135E-00	-1.0726E-00	-1.1506E-00	-1.2002E-00	-1.2002E-00	-1.6039E-00	-2.9574E-00	-3.6455E-00
-3.2479E-00	-1.9090E-00	-1.2993E-00	-1.0726E-00	-6.9696E-01	3.3045E-01	-2.0097E-01	-8.6012E-01	-2.1514E-01	-1.6394E-00
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3.3753E-01	7.5549E-01	9.8218E-01	6.9173E-01	1.4993E-00	1.3222E-00	1.0672E-00	3.8712E-01	2.2419E-01	1.8168E-01
4.2682E-01	9.2551E-01	1.0376E-01	1.0376E-01	1.4072E-00	9.8218E-01	4.5796E-01	7.2007E-01	6.9882E-01	4.3671E-01
2.8088E-01	1.637E-02	3.4524E-01	2.1356E-01	1.9031E-00	9.2551E-01	9.1134E-01	5.2880E-01	5.9964E-01	8.1925E-01
8.4759E-01	1.0174E-00	1.3846E-00	1.6056E-00	1.3647E-00	1.4497E-00	9.8218E-01	4.3671E-01	3.2336E-01	3.6587E-01
6.3508E-01	7.7674E-01	-2.4347E-01	5.0755E-01	3.8712E-01	3.5170E-01	4.2962E-01	1.2939E-00	1.3151E-00	1.1593E-00
9.1842E-01	5.9256E-01	4.7058E-02	3.7295E-01	1.6555E-01	-8.1761E-01	2.2419E-01	2.2502E-00	3.4462E-01	5.9964E-01
8.0508E-01	1.4781E-00	1.4781E-00	1.6906E-00	1.5985E-00	1.0955E-00	2.1002E-01	1.4001E-00	1.3860E-00	3.0920E-01
2.9503E-01	4.5534E-01	4.5534E-01	1.0672E-00	-2.2293E-01	5.4297E-01	1.8004E-00	2.3140E-00	1.5347E-00	1.3647E-00
2.1088E-00	2.2502E-00	2.3919E-00	2.2998E-00	2.0669E-00	1.4993E-00	1.1805E-00	2.5741E-00	1.8748E-00	1.6906E-00
1.7614E-00	1.5985E-00	1.7614E-00	1.7189E-00	1.8854E-00	3.9150E-00	1.1168E-00	1.1168E-00	1.3010E-00	6.4423E-01
-1.3013E-01	2.2419E-01	3.9420E-01	3.4462E-01	1.6751E-01	1.6043E-00	4.7757E-01	1.1026E-00	1.568E-00	1.4426E-00
7.6966E-01	1.4214E-00	1.3040E-00	1.2443E-00	9.5385E-01	5.5005E-01	2.3352E-00	2.6186E-00	1.8394E-00	1.9314E-00
2.1581E-00	2.2784E-00	1.4606E-00	1.3718E-00	1.5948E-00	5.1731E-01	4.7213E-01	-1.0743E-00	-2.2930E-01	-1.2710E-00
-3.4365E-01	-8.3886E-01	-1.4430E-01	4.5534E-03	-5.7653E-01	-1.4056E-00	-2.2065E-00	-1.5756E-00	5.0755E-01	3.1628E-01
2.5961E-01	5.5343E-03	4.5534E-03	9.7510E-01	1.0459E-00	7.2716E-01	-2.5307E-03	2.1002E-01	3.6587E-01	-1.0160E-00
-2.5891E-00	-5.3403E-01	-5.3403E-01	-2.7181E-01	-1.1860E-00	-2.5307E-03	1.9019E-00	2.1002E-01	4.6504E-01	-2.5307E-03
-4.9861E-01	6.0673E-01	6.0673E-01	6.9173E-01	3.5170E-01	1.1793E-01	2.8018E-01	4.7058E-02	1.2797E-00	1.5206E-00
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9.0262E-01	4.6504E-01	4.6504E-01	5.7653E-01	-1.6323E-01	-1.1293E-00	3.8004E-01	1.3918E-01	-7.8197E-01	-2.4347E-01
-9.4651E-02	1.8164E-01	8.7592E-01	4.5796E-01	2.5961E-01	3.4462E-01	2.4273E-00	-4.402E-01	-1.3064E-00	-8.0344E-01
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-4.3485E-01	-8.8137E-01	-2.0932E-00	-1.6677E-00	1.6043E-00	-1.2002E-00	-1.2002E-00	-1.7390E-00	-1.0939E-00	4.0837E-01
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3.0211E-01	6.9173E-01	5.9964E-01	9.3968E-01	1.6127E-00	1.3364E-00	4.923E-01	8.2633E-01	8.6175E-01	5.5714E-01
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3.8004E-01	1.0672E-00	1.2655E-00	1.1168E-00	8.0508E-01	-2.5056E-01	-1.2993E-00	-1.7673E-00	-1.5189E-00	-1.3914E-00
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6.9173E-01	1.1451E 00	3.9420E-01	2.5961E-01	1.4568E 00	1.1522E 00	6.1391E-01	7.9800E-01	-6.6315E-02
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1.5418E 00	1.8604E 00	1.7756E 00	1.5206E-01	1.2797E 00	4.1381E-01	-1.7263E-01	3.9008E 00	9.8218E-01
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4.7058E-02	9.6033E-01	6.9882E-01	1.0034E 00	3.5170E-01	1.1380E 00	1.5702E 00	1.2372E 00	-5.5288E-01
3.6587E-01	4.7740E-01	1.7972E-01	-1.9303E-01	-4.9507E-01	2.8066E-01	3.9420E-01	3.0211E-01	9.8927E-01
9.6093E-01	1.4710E 00	1.2868E 00	1.4851E 00	1.0105E 00	1.1664E 00	1.5702E 00	1.4285E 00	2.0660E 00
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6.6340E-01	1.2939E 00	1.5135E 00	1.1380E 00	8.9009E-01	-8.0483E-02	-1.6555E-01	-2.2222E-01	-2.4347E-01
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2.0023E 00	1.5372E 00	1.4515E 00	2.5805E-02	-1.6712E-02	-9.5929E-01	-1.4127E 00	-8.3178E-01	-1.1222E 00
1.7478E-01	2.9503E-01	1.1644E 00	1.7948E 00	1.9173E 00	1.2726E 00	4.6504E-01	8.5467E-01	6.7048E-01
5.7839E-01	7.2716E-01	-1.5846E-01	-2.7889E-01	4.1546E-01	-4.2777E-01	1.0318E 00	-6.7571E-01	5.2526E-01
7.2716E-01	2.5805E-02	-5.4420E-01	-1.1334E-01	-1.3347E 00	-1.4835E 00	-8.9545E-01	-6.7571E-01	-1.1293E 00
1.0844E-01	1.0844E-01	-1.7972E-01	-6.6843E-01	-2.2930E-01	-8.9545E-01	1.4035E 00	1.6039E 00	-8.6012E-01
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1.8168E-01	-2.5607E 00	-2.2845E 00	-1.2214E 00	-2.3057E 00	-3.0425E 00	-2.7074E 00	-1.6394E 00	-8.7567E-02
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5.0755E-01	5.4297E-01	3.5170E-01	3.0211E-01	3.1928E-01	4.6004E-01	6.8465E-01	4.5798E-01	7.0590E-01
3.1628E-01	7.9091E-01	5.7131E-01	1.1593E 00	1.2808E 00	4.6340E-01	4.0129E-01	1.6039E-01	-1.2304E-01
1.1637E-01	1.1637E-01	6.7757E-01	1.3918E-01	2.8086E-01	-2.0097E-01	1.4001E 00	6.6340E-01	3.6587E-01
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7.4132E-01	4.9338E-01	3.1628E-01	2.9503E-01	2.9503E-01	-2.5307E-03	6.4903E-01	5.3589E-01	6.8324E-02
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1.0376E-01	1.0376E-01	-1.0443E 00	-6.5466E-01	-5.4111E-01	2.3835E-01	-2.5076E-03	8.8300E-01	1.6043E-01
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2.2419E-01	7.0590E-01	7.2716E-01	6.1381E-01	-6.1381E-01	-6.737E-01	-6.4194E-01	-3.0895E-02	-4.5063E-02
-1.7972E-01	-1.6727E-02	-4.4902E-01	5.2172E-01	8.8300E-01	2.3127E-01	3.3045E-01	-1.2214E-00	-1.1506E-00
-7.4655E-01	-2.8014E-00	-2.0648E-01	-2.5395E-00	-2.3918E-00	-1.6677E-00	-1.6819E-00	-6.2612E-01	-5.1986E-01
-3.6401E-01	-1.5844E-01	-9.7346E-01	1.5827E-00	-1.5402E-01	-7.0405E-01	-4.7037E-01	-5.9070E-01	-1.0372E-00
-2.2222E-01	1.1805E-00	5.0755E-01	2.2006E-00	6.9173E-01	-9.5221E-01	-7.3947E-01	-4.7027E-01	-1.3135E-00
-2.9308E-01	-5.7653E-01	-3.4526E-01	1.1637E-02	3.5693E-01	2.1002E-01	-2.2089E-01	-1.0174E-01	-1.0174E-01
-1.6751E-01	-6.6315E-02	-1.5754E-00	1.4807E-00	-1.2002E-00	4.5534E-03	-2.3811E-02	-1.6394E-00	-5.6237E-01
-3.2844E-01	-8.5303E-01	-3.2140E-01	3.3556E-01	-7.749E-01	-1.5844E-01	-1.0160E-00	-1.6181E-00	-6.6863E-01
-5.8362E-01	2.6669E-01	7.0590E-01	1.631E-01	-3.1431E-01	-1.5402E-00	-1.0160E-00	-2.0097E-01	-5.6237E-01
-6.0487E-01	-1.1010E-00	1.1637E-02	4.2946E-01	8.1252E-01	1.2372E-00	5.6422E-01	8.0508E-01	1.8721E-02
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1.5499E-00	4.0837E-01	2.2502E-00	1.6197E-00	1.3718E-00	1.4851E-00	1.0672E-00	1.2018E-00	1.2443E-00
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-7.8906E-01	-8.0344E-01	1.0376E-01	1.0376E-01	-7.3947E-01	-7.6072E-01	-7.6072E-01	-7.5363E-01	-7.7489E-01	-7.6780E-01
-7.3947E-01	-7.1113E-01	-7.1113E-01	-7.0405E-01	-7.0405E-01	-6.9696E-01	-6.9696E-01	-6.9696E-01	-6.6863E-01	-6.6863E-01
-6.7571E-01	-6.7571E-01	-6.6154E-01	-6.8663E-01	-6.5446E-01	-6.2612E-01	-4.4029E-01	-6.2612E-01	-6.2612E-01	-6.2612E-01
-6.2612E-01	-6.2612E-01	-6.1195E-01	-5.9707E-01	-5.7653E-01	-5.8362E-01	-5.8362E-01	-5.9707E-01	-5.9707E-01	-5.9707E-01

APPENDIX B
EXPERIMENT B - RESULTS

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B-1 MATERIAL DATA AND PARAMETER SELECTION

B-1 MATERIAL DATA AND PARAMETER SELECTIONMaterial Data

Material	C1020
Manufacturing process	Grinding

Talysurf Set-Up

Sample length	10 mm
Vertical magnification	X 20000
Horizontal magnification	"AV"

Parameters for Data Processing

Number of data points	4000
Number of amplitude class interval	40
Maximum autocorrelation lag number	40
Standardized error for spectral Density	0.1

Data has been normalized and standardized.

TABLE B-2
THE PROBABILITY DENSITY AND DISTRIBUTION FUNCTIONS
COMPUTED FOR EXPERIMENT B

DISTRIBUTION OF AMPLITUDES.

CLASS	RANGE	MIDPOINT	HISTOGRAM	PERCENT	PROB. DENS.	PROB. DISTR.
0	-INFINITY TO -5.0000	-5.1250	1	.0003	.0010	.0003
1	-5.0000 TO -4.7500	-4.8750	0	0	0	.0003
2	-4.7500 TO -4.5000	-4.6250	0	0	0	.0003
3	-4.5000 TO -4.2500	-4.3750	1	.0003	.0010	.0005
4	-4.2500 TO -4.0000	-4.1250	0	0	0	.0005
5	-4.0000 TO -3.7500	-3.8750	5	.0013	.0050	.0018
6	-3.7500 TO -3.5000	-3.6250	7	.0018	.0070	.0035
7	-3.5000 TO -3.2500	-3.3750	5	.0013	.0050	.0048
8	-3.2500 TO -3.0000	-3.1250	9	.0023	.0090	.0070
9	-3.0000 TO -2.7500	-2.8750	25	.0063	.0251	.0133
10	-2.7500 TO -2.5000	-2.6250	19	.0048	.0191	.0181
11	-2.5000 TO -2.2500	-2.3750	28	.0070	.0282	.0251
12	-2.2500 TO -2.0000	-2.1250	43	.0108	.0432	.0359
13	-2.0000 TO -1.7500	-1.8750	55	.0138	.0553	.0498
14	-1.7500 TO -1.5000	-1.6250	74	.0186	.0744	.0684
15	-1.5000 TO -1.2500	-1.3750	108	.0271	.1086	.0955
16	-1.2500 TO -1.0000	-1.1250	170	.0427	.1709	.1383
17	-1.0000 TO -0.7500	-0.8750	225	.0566	.2262	.1948
18	-0.7500 TO -0.5000	-0.6250	321	.0807	.3228	.2755
19	-0.5000 TO -0.2500	-0.3750	354	.0890	.3560	.3645
20	-0.2500 TO 0	-0.1250	450	.1131	.4525	.4776
21	0 TO .2500	.1250	421	.1058	.4233	.5835
22	.2500 TO .5000	.3750	451	.1134	.4535	.6968
23	.5000 TO .7500	.6250	378	.0950	.3801	.7919
24	.7500 TO 1.0000	.8750	274	.0689	.2755	.8607
25	1.0000 TO 1.2500	1.1250	207	.0520	.2081	.9128
26	1.2500 TO 1.5000	1.3750	142	.0357	.1428	.9485
27	1.5000 TO 1.7500	1.6250	86	.0216	.0865	.9701
28	1.7500 TO 2.0000	1.8750	53	.0133	.0533	.9834
29	2.0000 TO 2.2500	2.1250	26	.0065	.0261	.9899
30	2.2500 TO 2.5000	2.3750	18	.0045	.0181	.9945
31	2.5000 TO 2.7500	2.6250	9	.0023	.0090	.9967
32	2.7500 TO 3.0000	2.8750	5	.0013	.0050	.9980
33	3.0000 TO 3.2500	3.1250	4	.0010	.0040	.9990
34	3.2500 TO 3.5000	3.3750	1	.0003	.0010	.9992
35	3.5000 TO 3.7500	3.6250	1	.0003	.0010	.9995
36	3.7500 TO 4.0000	3.8750	2	.0005	.0020	1.0000
37	4.0000 TO 4.2500	4.1250	0	0	0	1.0000
38	4.2500 TO 4.5000	4.3750	0	0	0	1.0000
39	4.5000 TO 4.7500	4.6250	0	0	0	1.0000
40	4.7500 TO 5.0000	4.8750	0	0	0	1.0000
41	5.0000 TO INFINITY	5.1250	0	0	0	1.0000

MINIMUM= -5.18272E 00
 MAXIMUM= 3.91498E 00
 NO OF CLASS AMPL.= 40
 CLASS INTERVAL SIZE= 2.50000E-01
 MAX LAG NO= 40
 STD.DEV.CORRECTION= 4.30792E-01

TABLE B-2 THE PROBABILITY DENSITY AND DISTRIBUTION FUNCTIONS COMPUTED FOR EXPERIMENT B

FIGURE B-3
THE PROBABILITY DENSITY FUNCTION PLOT FOR
EXPERIMENT B

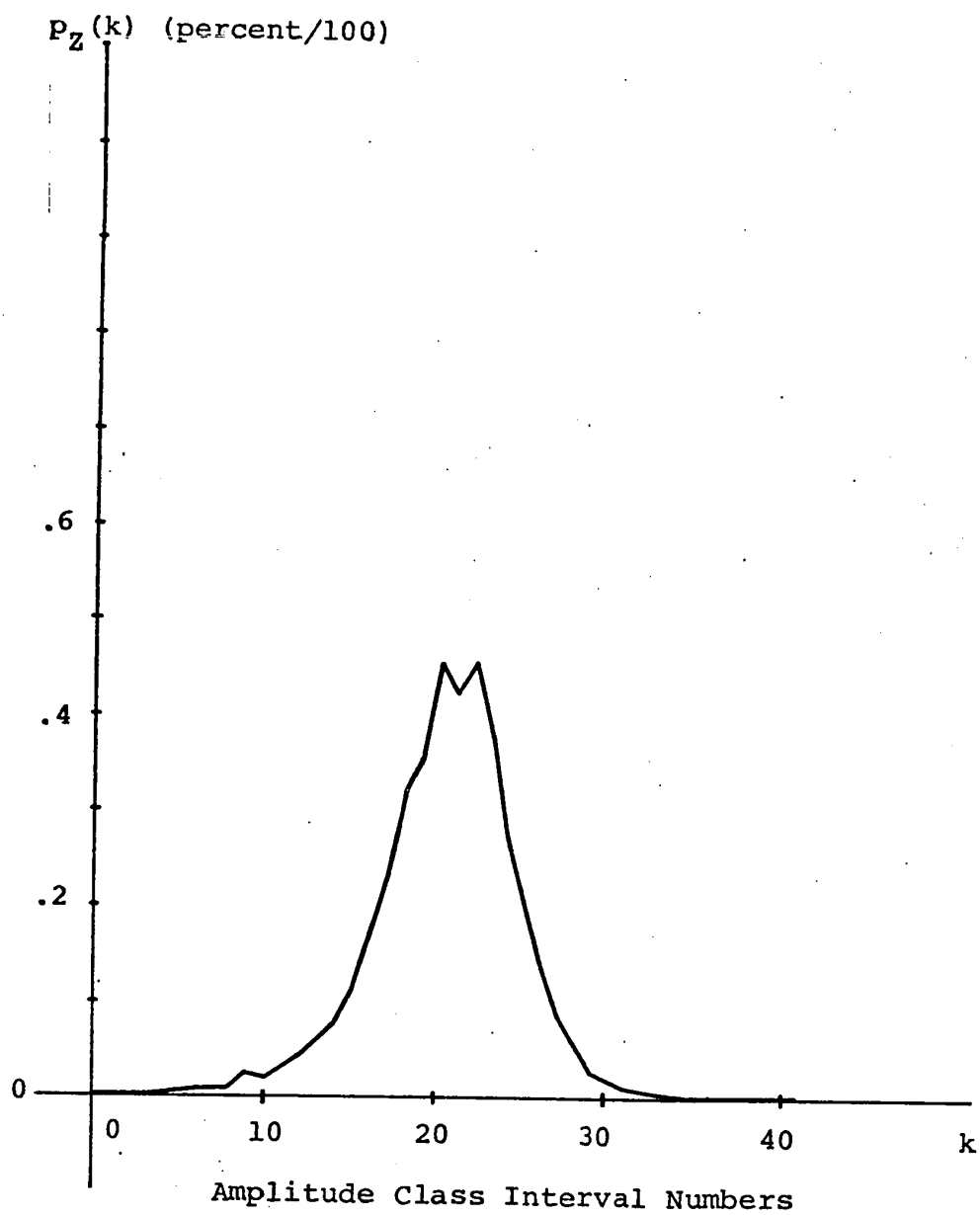


FIG. B-3 THE PROBABILITY DENSITY FUNCTION PLOT
FOR EXPERIMENT B (See also Table B-2)

$$k = 0.25; \quad \sigma_Y = 0.43 \mu\text{m}$$

FIGURE B-4
THE PROBABILITY DISTRIBUTION PLOT FOR
EXPERIMENT B

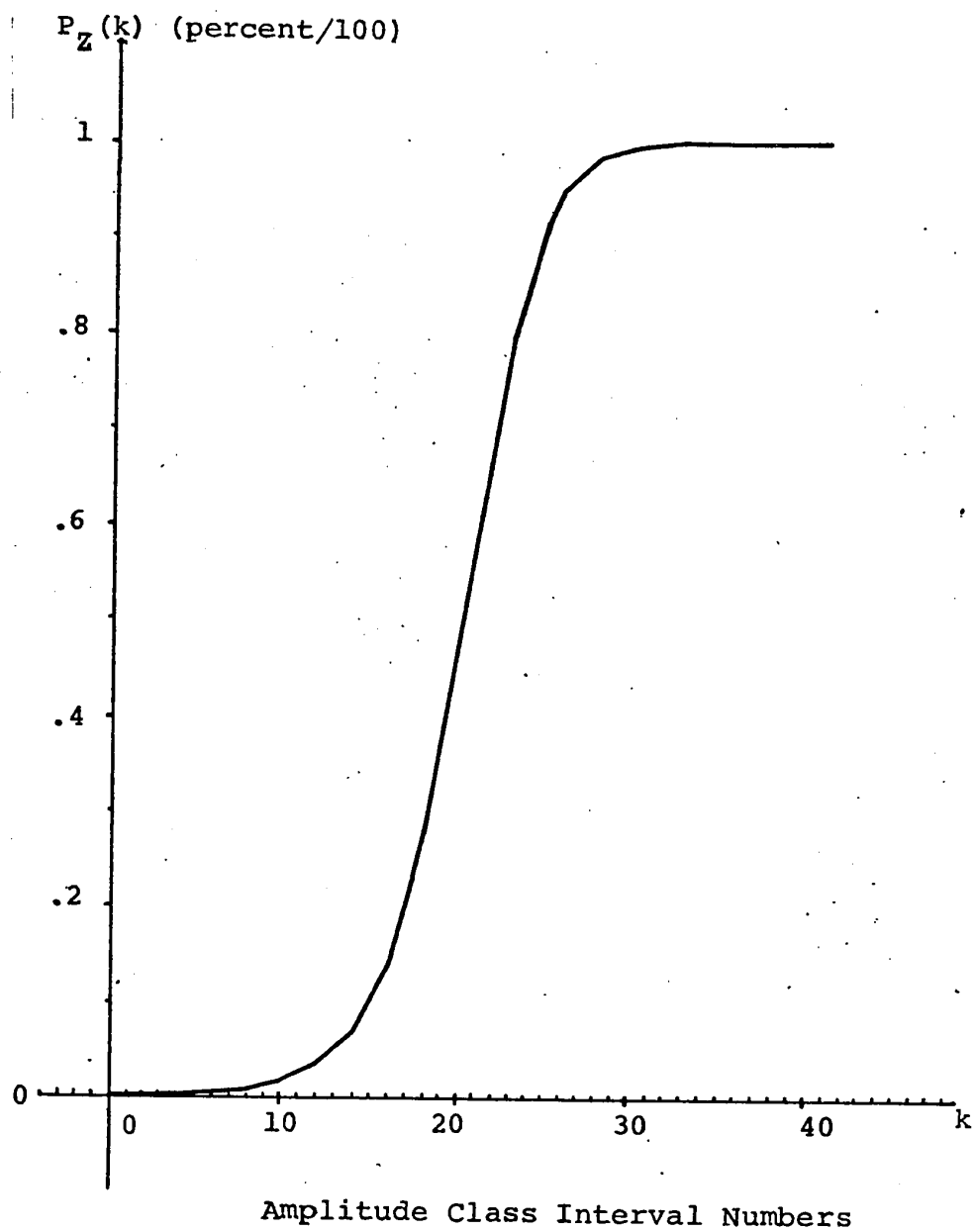


FIG.B-4 THE PROBABILITY DISTRIBUTION PLOT FOR
EXPERIMENT B (See also Table B-2)

$$k = 0.25; \quad \sigma_Y = 0.43 \mu\text{m}$$

TABLE B-5

**AUTOCORRELATION AND SPECTRAL DENSITY FUNCTIONS
COMPUTED FOR EXPERIMENT B**

AUTOCORRELATION AND SPECTRAL FUNCTIONS

LAG NO	AUTOCORRELATION	RAW SPECTRAL DENSITY	SMOOTH SPECTRAL DENSITY	FREQUENCY	WAVE LENGTH	HARM.NO.
0	9.9975E-01	5.0651E-02	3.6062E-02	0	0	0
1	7.5407E-01	2.1474E-02	2.7224E-02	7.6220E 00	1.9986E-01	1
2	5.8471E-01	1.5298E-02	1.5488E-02	1.5244E 01	9.9931E-02	2
3	4.8573E-01	9.8800E-03	1.0471E-02	2.2866E 01	6.6620E-02	3
4	4.0128E-01	6.8274E-03	7.7736E-03	3.0488E 01	4.9965E-02	4
5	3.4159E-01	7.5595E-03	6.9493E-03	3.8110E 01	3.9972E-02	5
6	3.0002E-01	5.8508E-03	5.7096E-03	4.5732E 01	3.3310E-02	6
7	2.5755E-01	3.5773E-03	4.1669E-03	5.3354E 01	2.8552E-02	7
8	2.3103E-01	3.6621E-03	3.4260E-03	6.0976E 01	2.4983E-02	8
9	2.2030E-01	2.8024E-03	2.9822E-03	6.8598E 01	2.2207E-02	9
10	2.1381E-01	2.6618E-03	2.5769E-03	7.6220E 01	1.9986E-02	10
11	1.9053E-01	2.1817E-03	2.2006E-03	8.3841E 01	1.8169E-02	11
12	1.7220E-01	1.7708E-03	1.8591E-03	9.1463E 01	1.6655E-02	12
13	1.7845E-01	1.6303E-03	1.7022E-03	9.9085E 01	1.5374E-02	13
14	1.6395E-01	1.1148E-03	1.5191E-03	1.0671E 02	1.4276E-02	14
15	1.4577E-01	1.5243E-03	1.3461E-03	1.1433E 02	1.3324E-02	15
16	1.3132E-01	1.8076E-03	1.4928E-03	1.2195E 02	1.2491E-02	16
17	1.3064E-01	9.4313E-04	1.5207E-03	1.2957E 02	1.1757E-02	17
18	1.1810E-01	7.4836E-04	1.1492E-03	1.3720E 02	1.1103E-02	18
19	9.5393E-02	6.4103E-04	8.7443E-04	1.4482E 02	1.0519E-02	19
20	8.7793E-02	8.1204E-04	7.6022E-04	1.5244E 02	9.9931E-03	20
21	8.1556E-02	9.1435E-04	9.0008E-04	1.6066E 02	9.5172E-03	21
22	9.3227E-02	7.4756E-04	8.4708E-04	1.6768E 02	9.0846E-03	22
23	8.6611E-02	8.8524E-04	8.2368E-04	1.7530E 02	8.6896E-03	23
24	9.3232E-02	6.2922E-04	7.8682E-04	1.8293E 02	8.3276E-03	24
25	8.9379E-02	5.5387E-04	6.7439E-04	1.9055E 02	7.9945E-03	25
26	9.5706E-02	5.9474E-04	5.8293E-04	1.9817E 02	7.6870E-03	26
27	1.1390E-01	6.5462E-04	6.3672E-04	2.0579E 02	7.4023E-03	27
28	1.2320E-01	4.8767E-04	5.9803E-04	2.1341E 02	7.1379E-03	28
29	1.1796E-01	4.8737E-04	6.1134E-04	2.2104E 02	6.8918E-03	29
30	1.0900E-01	4.0296E-04	5.0808E-04	2.2866E 02	6.6620E-03	30
31	1.0542E-01	4.7241E-04	4.4143E-04	2.3628E 02	6.4471E-03	31
32	1.1056E-01	4.2278E-04	4.264E-04	2.4390E 02	6.2457E-03	32
33	8.9446E-02	4.2938E-04	4.3684E-04	2.5152E 02	6.0564E-03	33
34	9.1853E-02	4.4584E-04	4.3184E-04	2.5915E 02	5.8783E-03	34
35	9.6994E-02	4.5657E-04	4.441E-04	2.6677E 02	5.7103E-03	35
36	9.4067E-02	4.5761E-04	4.5414E-04	2.7439E 02	5.5517E-03	36
37	9.7933E-02	1.0326E-01	4.3643E-04	2.8201E 02	5.4017E-03	37
38			4.1525E-04	2.8963E 02	5.2595E-03	38
39				2.9726E 02	5.1246E-03	39
40				3.0488E 02	4.9965E-03	40

TEST FOR RAW SP.DENS=7.4069249E-09

NORMALISED AND STANDARDISED

MINIMUM=-5.18272E 00
 MAXIMUM=3.91498E 00
 NO OF POINTS REC.=3978
 NO OF CLASS AMPL.=40
 MAX.LAG NO=40
 STD.DEV.CORRECTION=4.30792E-01

TABLE B-5
AUTOCORRELATION AND SPECTRAL DENSITY FUNCTIONS
COMPUTED FOR EXPERIMENT B

FIGURE B-6
THE AUTOCORRELATION FUNCTION PLOT FOR
EXPERIMENT B

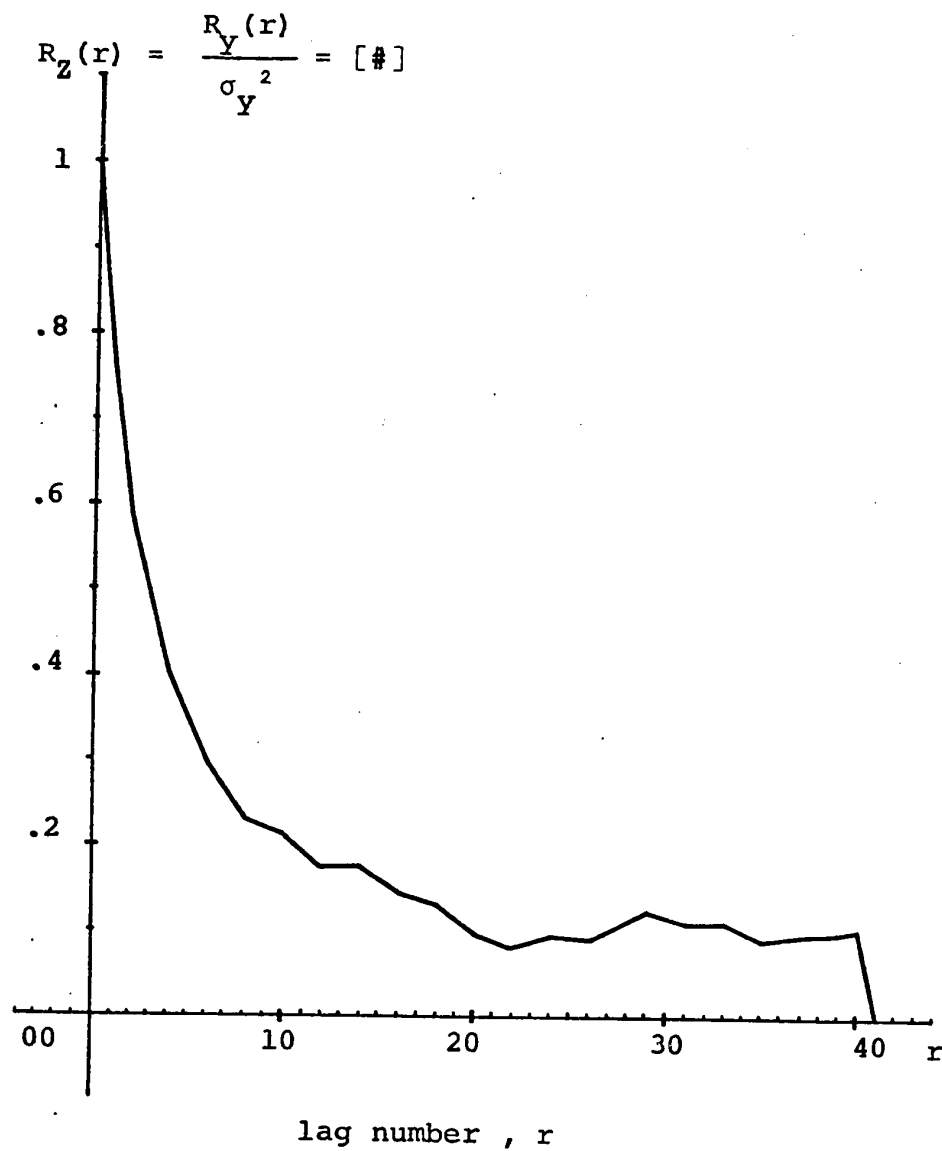


FIG. B-6 THE AUTOCORRELATION FUNCTION PLOT FOR
EXPERIMENT B (See also Table B-5)

$$r = 2.5 \mu\text{m}; \quad \sigma_Y = 0.43 \mu\text{m}$$

FIGURE B-7
THE SPECTRAL DENSITY FUNCTION PLOT FOR
EXPERIMENT B

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Figure

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C-4	The probability distribution plot for Experiment C.	C-5
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C-7	The spectral density function plot for Experiment C.	C-8

Table

C-2	The probability density and distribution functions computed for Experiment C	C-3
C-5	Autocorrelation and spectral density functions computed for Experiment C. . .	C-6

C-1 MATERIAL DATA AND PARAMETER SELECTION

C-1 MATERIAL, DATA AND PARAMETER SELECTIONMaterial Data

Material	C1020
Manufacturing process	Grinding

Talysurf Set-Up

Sample length	5 mm
Vertical magnification	X 20000
Horizontal magnification	"AV"

Parameters for Data Processing

Number of data points	2000
Number of amplitude class interval	40
Maximum autocorrelation lag number	40
Standardized error for spectral Density	.14

Data has been normalized and standardized

TABLE C-2
THE PROBABILITY DENSITY AND DISTRIBUTION FUNCTIONS
COMPUTED FOR EXPERIMENT C

DISTRIBUTION OF AMPLITUDES.

CLASS	RANGE	MIDPOINT	HISTOGRAM	PERCENT	PROB. DENS.	PROB. DISTR.
0	-INFINITY TO -5.0000	-5.1250	0	0	0	0
1	-5.0000 TO -4.7500	-4.8750	0	0	0	0
2	-4.7500 TO -4.5000	-4.6250	0	0	0	0
3	-4.5000 TO -4.2500	-4.3750	0	0	0	0
4	-4.2500 TO -4.0000	-4.1250	0	0	0	0
5	-4.0000 TO -3.7500	-3.8750	0	0	0	0
6	-3.7500 TO -3.5000	-3.6250	2	.0010	.0040	.0010
7	-3.5000 TO -3.2500	-3.3750	2	.0010	.0040	.0020
8	-3.2500 TO -3.0000	-3.1250	7	.0035	.0140	.0055
9	-3.0000 TO -2.7500	-2.8750	17	.0085	.0340	.0140
10	-2.7500 TO -2.5000	-2.6250	6	.0030	.0120	.0170
11	-2.5000 TO -2.2500	-2.3750	19	.0095	.0380	.0265
12	-2.2500 TO -2.0000	-2.1250	28	.0140	.0560	.0405
13	-2.0000 TO -1.7500	-1.8750	30	.0150	.0600	.0555
14	-1.7500 TO -1.5000	-1.6250	36	.0180	.0720	.0735
15	-1.5000 TO -1.2500	-1.3750	56	.0280	.1120	.1015
16	-1.2500 TO -1.0000	-1.1250	82	.0410	.1640	.1425
17	-1.0000 TO -0.7500	-0.8750	99	.0495	.1980	.1920
18	-0.7500 TO -0.5000	-0.6250	147	.0735	.2940	.2655
19	-0.5000 TO -0.2500	-0.3750	181	.0905	.3620	.3560
20	-0.2500 TO 0	-0.1250	230	.1150	.4600	.4710
21	0 TO .2500	.1250	221	.1105	.4420	.5815
22	.2500 TO .5000	.3750	239	.1195	.4780	.7010
23	.5000 TO .7500	.6250	187	.0935	.3740	.7945
24	.7500 TO 1.0000	.8750	151	.0755	.3020	.8700
25	1.0000 TO 1.2500	1.1250	86	.0430	.1720	.9130
26	1.2500 TO 1.5000	1.3750	73	.0365	.1460	.9495
27	1.5000 TO 1.7500	1.6250	40	.0200	.0800	.9695
28	1.7500 TO 2.0000	1.8750	23	.0115	.0460	.9810
29	2.0000 TO 2.2500	2.1250	14	.0070	.0280	.9880
30	2.2500 TO 2.5000	2.3750	12	.0060	.0240	.9940
31	2.5000 TO 2.7500	2.6250	6	.0030	.0120	.9970
32	2.7500 TO 3.0000	2.8750	2	.0010	.0040	.9980
33	3.0000 TO 3.2500	3.1250	1	.0005	.0020	.9985
34	3.2500 TO 3.5000	3.3750	1	.0005	.0020	.9990
35	3.5000 TO 3.7500	3.6250	1	.0005	.0020	.9995
36	3.7500 TO 4.0000	3.8750	1	.0005	.0020	1.0000
37	4.0000 TO 4.2500	4.1250	0	0	0	1.0000
38	4.2500 TO 4.5000	4.3750	0	0	0	1.0000
39	4.5000 TO 4.7500	4.6250	0	0	0	1.0000
40	4.7500 TO 5.0000	4.8750	0	0	0	1.0000
41	5.0000 TO +INFINITY	5.1250	0	0	0	1.0000

MINIMUM= -3.72643E 00
 MAXIMUM= 3.89179E 00
 NO OF CLASS AMPL.= 40
 CLASS INTERVAL SIZE= 2.50000E-01
 MAX.LAG NO= 40
 STD.DEV.CORRECTION= 4.33134E-01

TABLE C-2 THE PROBABILITY DENSITY AND DISTRIBUTION FUNCTIONS COMPUTED FOR
EXPERIMENT C

FIGURE C-3
THE PROBABILITY DENSITY FUNCTION PLOT FOR
EXPERIMENT C

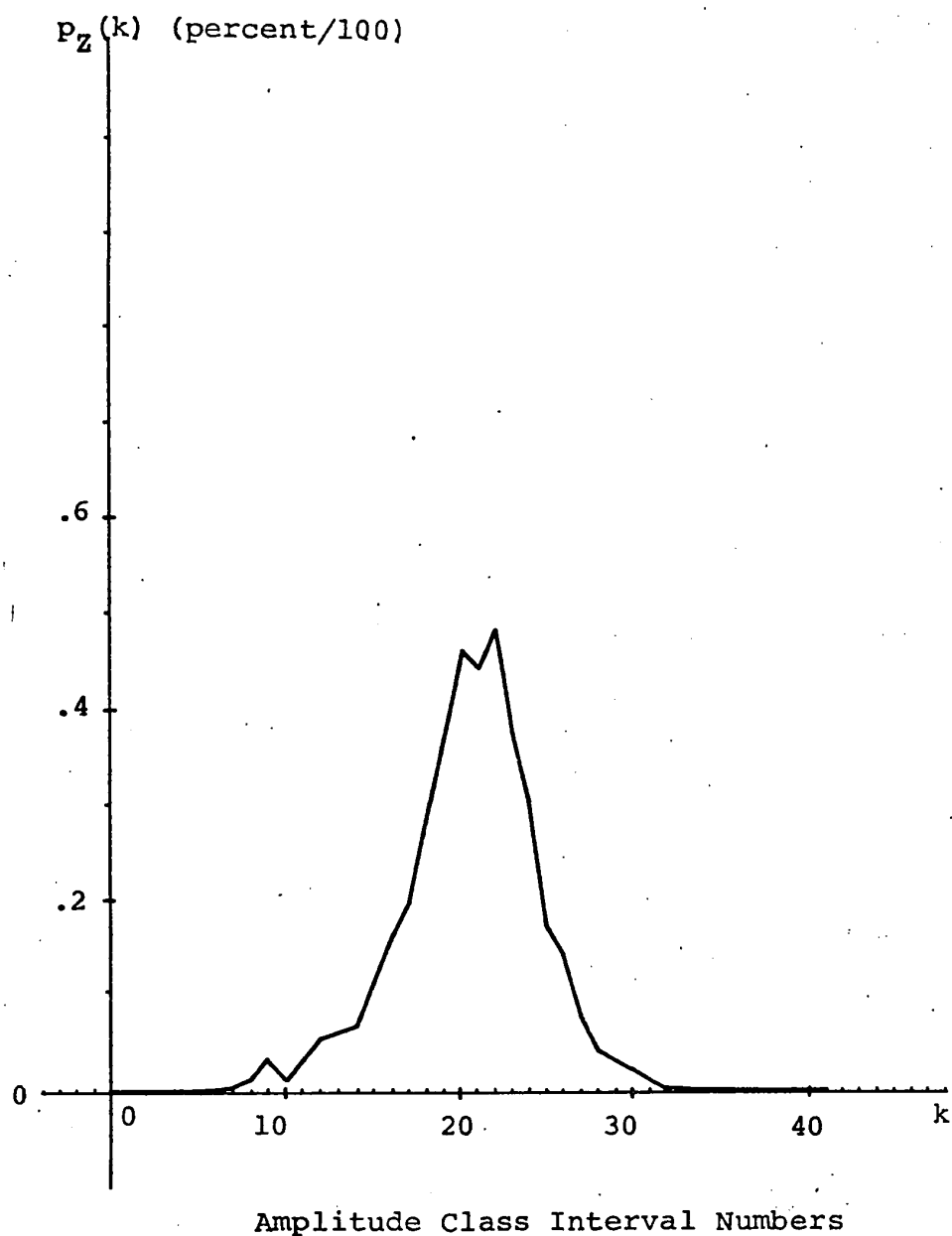


FIG. C-3 THE PROBABILITY DENSITY FUNCTION PLOT FOR EXPERIMENT C (See also Table C-2)

$$k = 0.25; \quad \sigma_y = 0.433 \mu\text{m}$$

FIGURE C-4
THE PROBABILITY DISTRIBUTION PLOT FOR
EXPERIMENT C

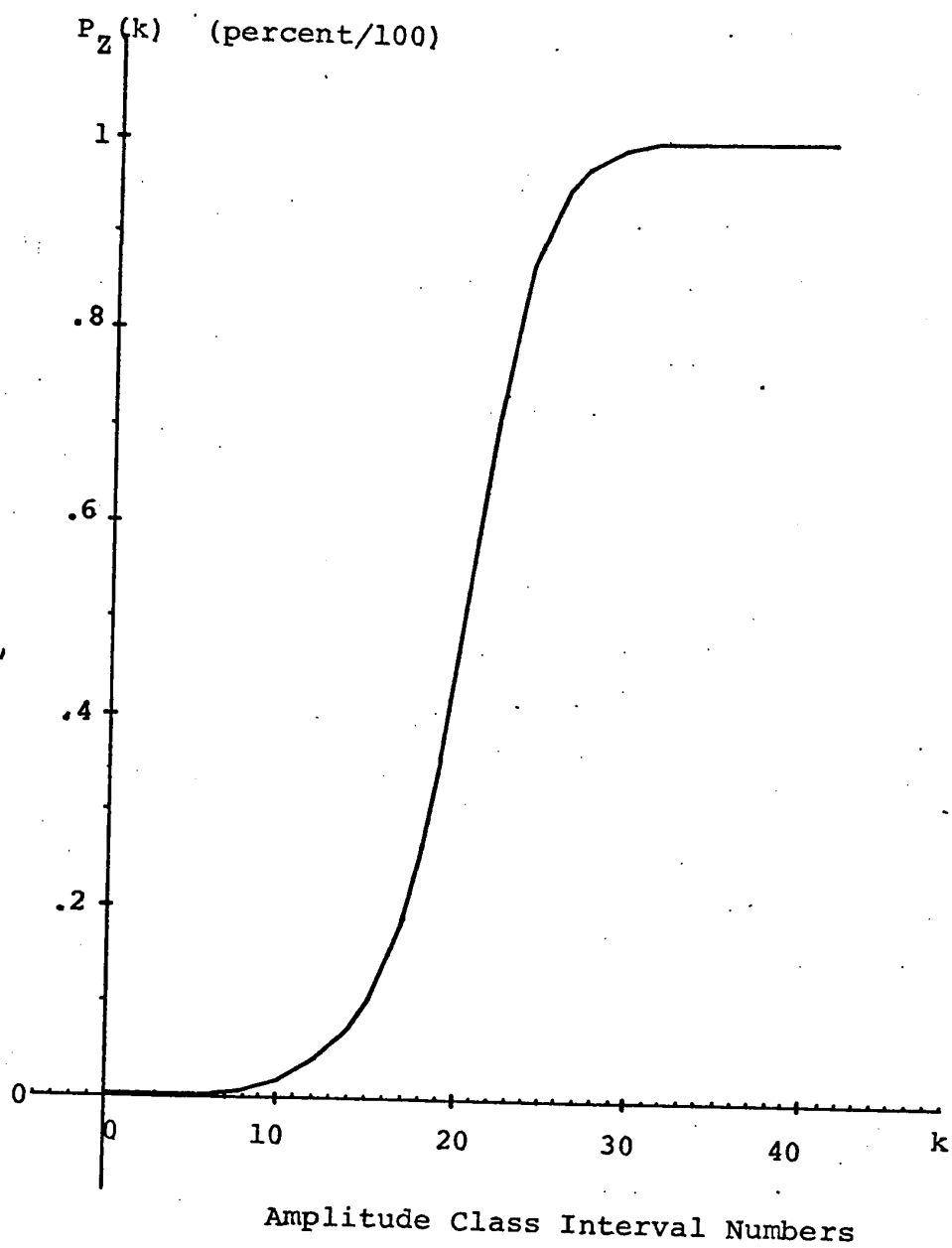


FIG. C-4 THE PROBABILITY DISTRIBUTION PLOT FOR EXPERIMENT C (See also Table C-2)

$$k = 0.25;$$

$$\sigma_y = 0.433 \mu\text{m}$$

TABLE C-5

AUTOCORRELATION AND SPECTRAL DENSITY FUNCTIONS
COMPUTED FOR EXPERIMENT C

AUTOCORRELATION AND SPECTRAL FUNCTIONS						
LAG NO	AUTOCORRELATION	RAW SPECTRAL DENSITY	SMOOTH SPECTRAL DENSITY	FREQUENCY	AVERAGE LENGTH	HARMONIC
0	9.9950E-01	3.5362E-02	2.7919E-02	7.6220E-00	1.9986E-01	0
1	7.4153E-01	2.0475E-02	2.3961E-02	1.5244E-01	9.9931E-02	1
2	5.5907E-01	1.9531E-02	1.7314E-02	2.2866E-01	6.6620E-02	2
3	4.5611E-01	9.7196E-03	1.1440E-02	3.0488E-01	4.9465E-02	3
4	3.5535E-01	6.7877E-03	8.2529E-03	3.8110E-01	3.9972E-02	4
5	2.8632E-01	9.7167E-03	8.1205E-03	4.5732E-01	3.3310E-02	5
6	2.3089E-01	6.2609E-03	6.5645E-03	5.3354E-01	2.8552E-02	6
7	1.8157E-01	4.0194E-03	4.6238E-03	6.0976E-01	2.4983E-02	7
8	1.5166E-01	4.1955E-03	3.9454E-03	6.8948E-01	2.2207E-02	8
9	1.4498E-01	3.3714E-03	3.3372E-03	7.6220E-01	2.0076E-02	9
10	1.5531E-01	2.4105E-03	2.4812E-03	8.3841E-01	1.8169E-02	10
11	1.4099E-01	1.7324E-03	1.8066E-03	9.1463E-01	1.6555E-02	11
12	1.0979E-01	1.3511E-03	1.5561E-03	9.9085E-01	1.5374E-02	12
13	9.1726E-02	1.7899E-03	1.8801E-03	1.0671E-02	1.4276E-02	13
14	8.0206E-02	2.2000E-03	1.8012E-03	1.1433E-02	1.3324E-02	14
15	7.8836E-02	1.6874E-03	1.5536E-03	1.2195E-02	1.2491E-02	15
16	6.8195E-02	1.3307E-03	1.4325E-03	1.2957E-02	1.1757E-02	16
17	5.4867E-02	1.0245E-03	1.1318E-03	1.3720E-02	1.1103E-02	17
18	3.2687E-02	9.6916E-04	9.4212E-04	1.4482E-02	1.0519E-02	18
19	3.3536E-03	8.0570E-04	8.1413E-04	1.5244E-02	9.9931E-03	19
20	-1.3295E-02	6.7597E-04	8.3465E-04	1.6006E-02	9.5172E-03	20
21	-1.9853E-02	1.1810E-03	9.4934E-04	1.6768E-02	9.0846E-03	21
22	-1.2687E-02	7.5945E-04	9.7228E-04	1.7530E-02	8.6896E-03	22
23	-9.5959E-03	1.1893E-03	9.9954E-04	1.8293E-02	8.3276E-03	23
24	-2.2216E-03	8.6019E-04	9.1772E-04	1.9055E-02	7.9945E-03	24
25	8.3933E-03	7.6124E-04	7.4654E-04	1.9817E-02	7.6870E-03	25
26	2.2729E-02	6.0347E-04	6.4917E-04	2.0579E-02	7.4023E-03	26
27	4.7339E-02	6.2850E-04	6.4347E-04	2.1341E-02	7.1379E-03	27
28	7.4737E-02	7.1339E-04	7.0168E-04	2.2104E-02	6.8918E-03	28
29	7.8884E-02	7.5144E-04	7.3345E-04	2.2866E-02	6.6620E-03	29
30	7.2446E-02	7.1753E-04	6.7985E-04	2.3628E-02	6.4471E-03	30
31	7.1479E-02	5.3292E-04	5.4389E-04	2.4390E-02	6.2457E-03	31
32	7.8486E-02	3.9217E-04	4.4819E-04	2.5152E-02	6.0564E-03	32
33	7.8459E-02	4.7551E-04	4.380E-04	2.5915E-02	5.8783E-03	33
34	7.1711E-02	4.3201E-04	4.5944E-04	2.6677E-02	5.7103E-03	34
35	7.3833E-02	4.9823E-04	4.5312E-04	2.7439E-02	5.5517E-03	35
36	6.7892E-02	3.8401E-04	4.3381E-04	2.8201E-02	5.4017E-03	36
37	5.9576E-02	4.6901E-04	4.2852E-04	2.8963E-02	5.2595E-03	37
38	6.3977E-02	3.9203E-04	3.8727E-04	2.9726E-02	5.1246E-03	38
39	7.6277E-02	2.9598E-04	3.8250E-04	3.0488E-02	4.9965E-03	39
40						40

TEST FOR RAW SP.DENS=7.2759576E-09

NORMALISED AND STANDARDISED

MINIMUM= -3.72643E 00
 MAXIMUM= 3.89179E 00
 NO OF POINTS REC.= 2000
 NO OF CLASS AMPL.= 40
 MAX LAG NO= 40
 STD.DEV.CORRECTION= 4.33134E-01

TABLE C-5 AUTOCORRELATION AND SPECTRAL DENSITY FUNCTIONS
COMPUTED FOR EXPERIMENT C

FIGURE C-6
THE AUTOCORRELATION FUNCTION PLOT FOR
EXPERIMENT C

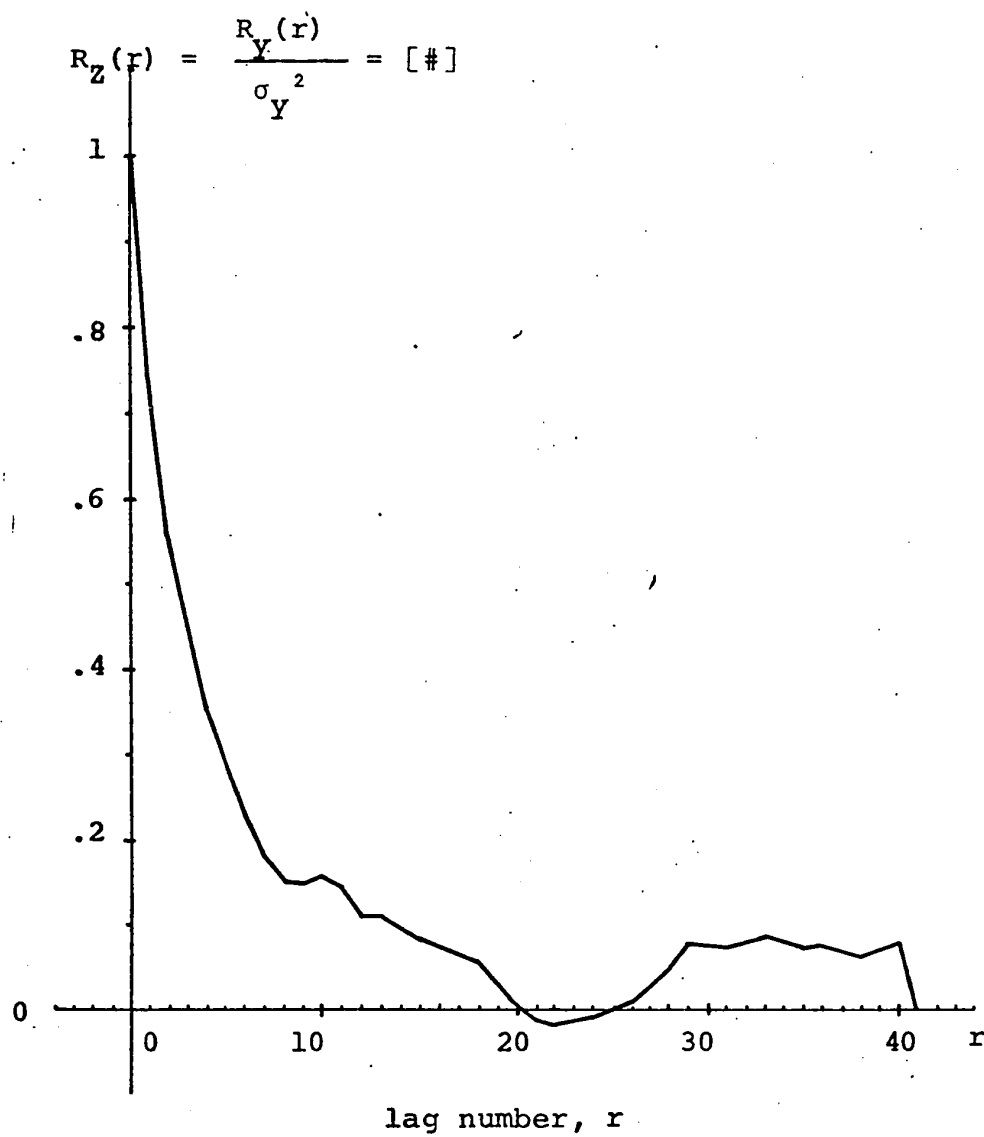


FIG. C-6 THE AUTOCORRELATION FUNCTION PLOT FOR
EXPERIMENT C (See also Table C-5)

$$r = 2.5 \mu\text{m}; \quad \sigma_Y = 0.433 \mu\text{m}$$

FIGURE C-7
THE SPECTRAL DENSITY FUNCTION PLOT FOR
EXPERIMENT C

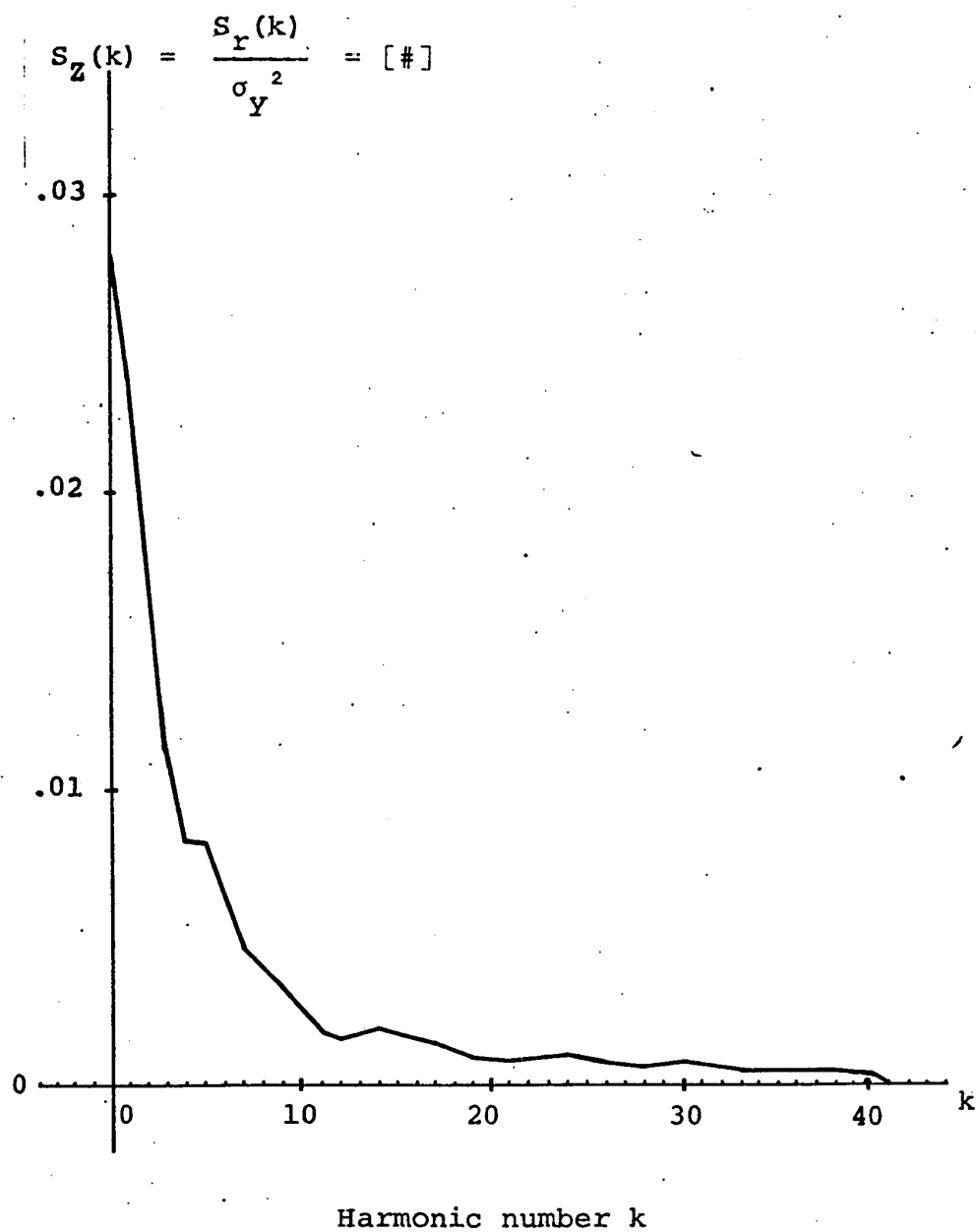


FIG. C-7 THE SPECTRAL DENSITY FUNCTION PLOT FOR EXPERIMENT C (See also Table C-5)

$\sigma_y = 0.433 \mu\text{m}; B_e = 15.25 \text{ cps}$

APPENDIX D
EXPERIMENT D - RESULTS

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D-1 MATERIAL DATA AND PARAMETER SELECTION

D-1 MATERIAL DATA AND PARAMETER SELECTIONMaterial Data

Material	C1020
Manufacturing process	Grinding

Talysurf Set-Up

Sample length	2.5 mm
Vertical magnification	X 20000
Horizontal magnification	"AV"

Parameters for Data Processing

Number of data points	1000
Number of amplitude class interval	40
Maximum autocorrelation lag number	40
Standardized error for spectral Density	.20

Data has been normalized and standardized

TABLE D-2

THE PROBABILITY DENSITY AND DISTRIBUTION FUNCTIONS

COMPUTED FOR EXPERIMENT D

DISTRIBUTION OF AMPLITUDES.

CLASS	RANGE	MIDPOINT	HISTOGRAM	PERCENT	PROB. DENS.	PROB. DISTR.
0	-INFINITY TO -5.0000	-5.1250	0	0	0	0
1	-5.0000 TO -4.7500	-4.8750	0	0	0	0
2	-4.7500 TO -4.5000	-4.6250	0	0	0	0
3	-4.5000 TO -4.2500	-4.3750	0	0	0	0
4	-4.2500 TO -4.0000	-4.1250	0	0	0	0
5	-4.0000 TO -3.7500	-3.8750	0	0	0	0
6	-3.7500 TO -3.5000	-3.6250	0	0	0	0
7	-3.5000 TO -3.2500	-3.3750	3	.0030	.0120	.0030
8	-3.2500 TO -3.0000	-3.1250	8	.0080	.0320	.0110
9	-3.0000 TO -2.7500	-2.8750	4	.0040	.0160	.0150
10	-2.7500 TO -2.5000	-2.6250	10	.0100	.0400	.0250
11	-2.5000 TO -2.2500	-2.3750	17	.0070	.0280	.0320
12	-2.2500 TO -2.0000	-2.1250	7	.0070	.0280	.0390
13	-2.0000 TO -1.7500	-1.8750	9	.0090	.0360	.0480
14	-1.7500 TO -1.5000	-1.6250	18	.0180	.0720	.0660
15	-1.5000 TO -1.2500	-1.3750	37	.0370	.1480	.1030
16	-1.2500 TO -1.0000	-1.1250	35	.0350	.1400	.1380
17	-1.0000 TO -0.7500	-0.8750	50	.0500	.2000	.1880
18	-0.7500 TO -0.5000	-0.6250	80	.0800	.3200	.2680
19	-0.5000 TO -0.2500	-0.3750	90	.0900	.3600	.3580
20	-0.2500 TO 0	-0.1250	110	.1100	.4400	.4680
21	0 TO .2500	.1250	99	.0990	.3960	.5670
22	.2500 TO .5000	.3750	132	.1320	.5280	.6990
23	.5000 TO .7500	.6250	197	.0970	.3880	.7960
24	.7500 TO 1.0000	.8750	78	.0780	.3120	.8740
25	1.0000 TO 1.2500	1.1250	48	.0480	.1920	.9220
26	1.2500 TO 1.5000	1.3750	30	.0300	.1200	.9520
27	1.5000 TO 1.7500	1.6250	19	.0190	.0760	.9710
28	1.7500 TO 2.0000	1.8750	11	.0110	.0440	.9820
29	2.0000 TO 2.2500	2.1250	8	.0080	.0320	.9900
30	2.2500 TO 2.5000	2.3750	4	.0040	.0160	.9940
31	2.5000 TO 2.7500	2.6250	3	.0030	.0120	.9970
32	2.7500 TO 3.0000	2.8750	1	.0010	.0040	.9980
33	3.0000 TO 3.2500	3.1250	1	.0010	.0040	.9990
34	3.2500 TO 3.5000	3.3750	1	.0010	.0040	1.0000
35	3.5000 TO 3.7500	3.6250	0	0	0	1.0000
36	3.7500 TO 4.0000	3.8750	0	0	0	1.0000
37	4.0000 TO 4.2500	4.1250	0	0	0	1.0000
38	4.2500 TO 4.5000	4.3750	0	0	0	1.0000
39	4.5000 TO 4.7500	4.6250	0	0	0	1.0000
40	4.7500 TO 5.0000	4.8750	0	0	0	1.0000
41	5.0000 TO +INFINITY	5.1250	0	0	0	1.0000

MINIMUM= -3.42854E 00
 MAXIMUM= 3.36998E 00
 NO OF CLASS AMPL.= 40
 CLASS INTERVAL SIZE= 2.50000E-01
 MAX.LAG NO= 40
 STD.DEV.CORRECTION= 4.07195E-01

TABLE D-2 THE PROBABILITY DENSITY AND DISTRIBUTION FUNCTIONS COMPUTED FOR EXPERIMENT D

FIGURE D-3
THE PROBABILITY DENSITY FUNCTION PLOT FOR
EXPERIMENT D

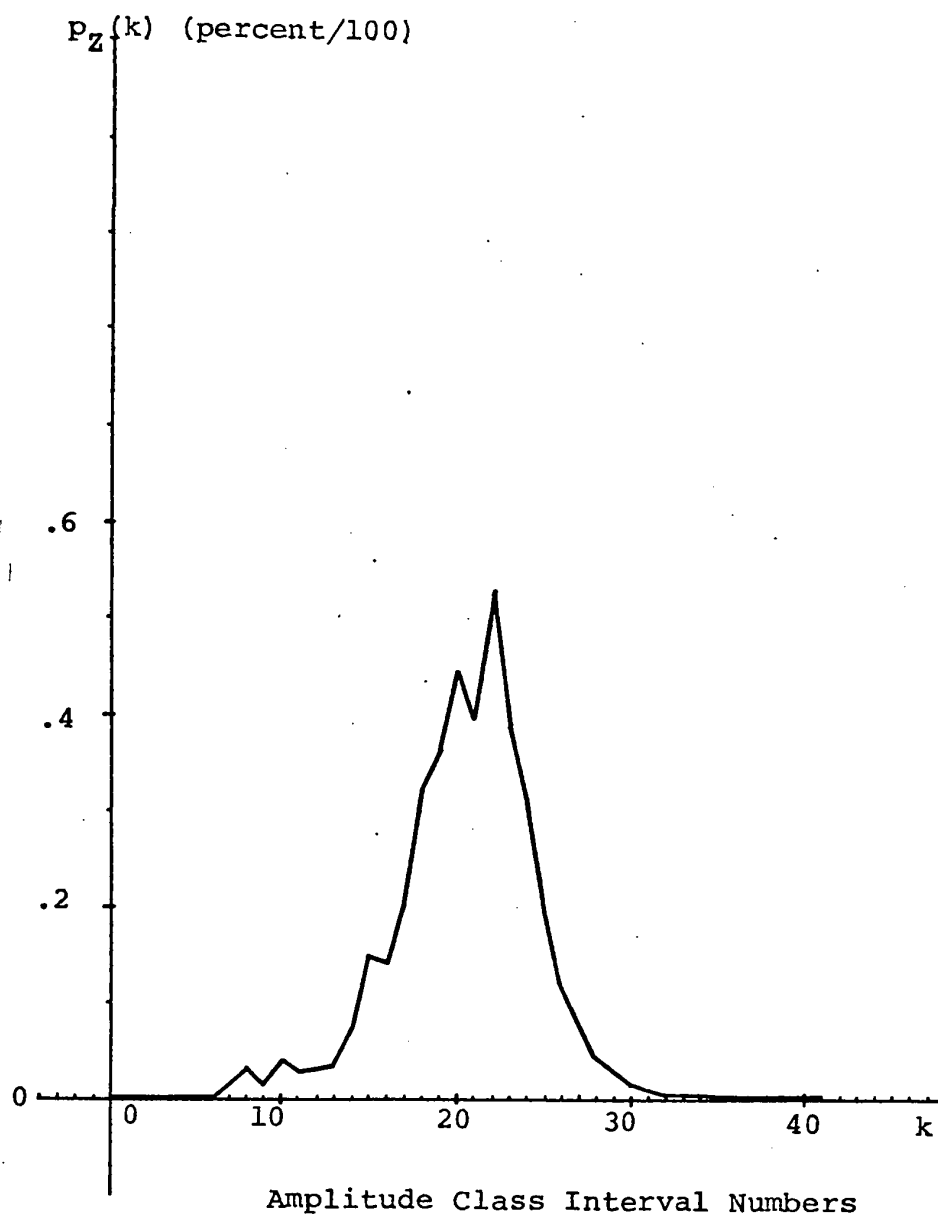


FIG. D-3 THE PROBABILITY DENSITY FUNCTION PLOT FOR EXPERIMENT D (See also Table D-2)

$$k = 0.25; \quad \sigma_y = 0.407 \mu\text{m}$$

FIGURE D-4
THE PROBABILITY DISTRIBUTION PLOT FOR
EXPERIMENT D

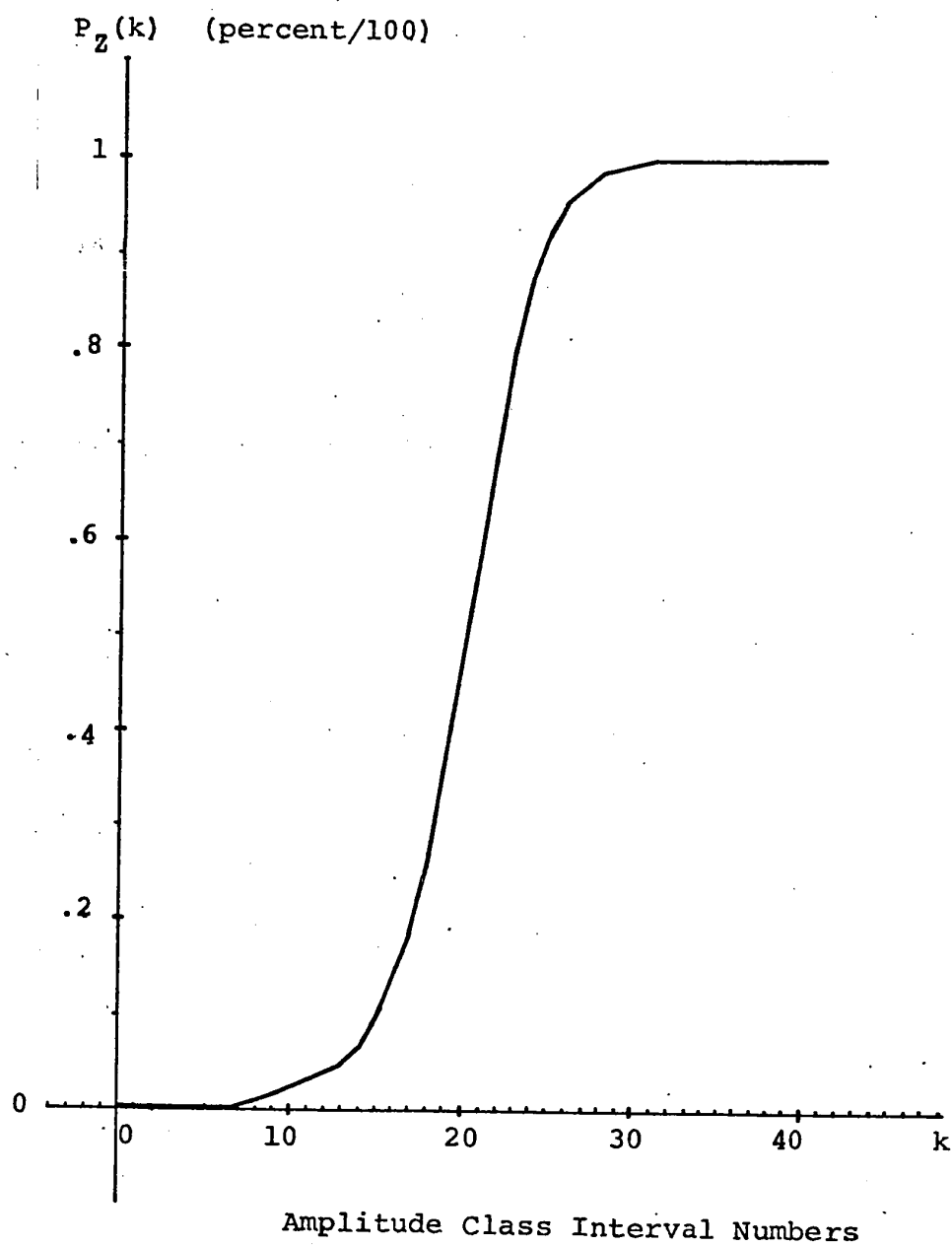


FIG. D-4 THE PROBABILITY DISTRIBUTION PLOT FOR EXPERIMENT D (See also Table D-2)

$$k = 0.25; \quad \sigma_Y = 0.407 \mu\text{m}$$

TABLE D-5
AUTOCORRELATION AND SPECTRAL DENSITY FUNCTIONS
COMPUTED FOR EXPERIMENT D

AUTOCORRELATION AND SPECTRAL FUNCTIONS

LAG NO	AUTOCORRELATION	RAW SPECTRAL DENSITY	SMOOTH SPECTRAL DENSITY	FREQUENCY	AVERAGE LENGTH	HARM. NO.
0	9.9900E-01	3.2396E-02	2.6186E-02	0	0	0
1	7.3132E-01	1.9776E-02	2.1311E-02	7.6220E 00	1.9986E-01	1
2	5.4584E-01	1.3296E-02	1.5353E-02	1.5244E 01	9.9931E-02	2
3	4.1874E-01	1.1047E-02	1.0803E-02	2.2866E 01	6.6620E-02	3
4	2.9438E-01	7.8223E-03	9.7822E-03	3.0488E 01	4.9965E-02	4
5	2.2060E-01	1.2437E-02	1.0391E-02	3.8110E 01	3.9972E-02	5
6	1.5313E-01	8.4690E-03	8.7549E-03	4.5732E 01	3.3310E-02	6
7	9.5829E-02	4.8450E-03	5.5638E-03	5.3354E 01	2.8552E-02	7
8	7.6203E-02	3.6964E-03	3.9388E-03	6.0976E 01	2.4983E-02	8
9	7.6587E-02	3.5176E-03	3.1788E-03	6.8598E 01	2.2207E-02	9
10	1.0475E-01	1.9836E-03	2.3009E-03	7.6220E 01	1.9986E-02	10
11	1.2277E-01	1.7189E-03	1.8444E-03	8.3841E 01	1.8189E-02	11
12	1.1776E-01	1.9565E-03	1.8540E-03	9.1463E 01	1.6655E-02	12
13	1.3907E-01	1.7842E-03	1.8115E-03	9.9085E 01	1.5374E-02	13
14	1.2733E-01	1.7211E-03	1.7976E-03	1.0671E 02	1.4276E-02	14
15	1.2353E-01	1.9641E-03	1.8135E-03	1.1433E 02	1.3324E-02	15
16	1.2915E-01	1.6047E-03	1.5027E-03	1.2195E 02	1.2491E-02	16
17	9.8511E-02	1.2373E-03	1.2568E-03	1.2957E 02	1.1757E-02	17
18	7.7526E-02	9.4786E-04	1.0038E-03	1.3720E 02	1.1103E-02	18
19	5.7674E-02	8.8204E-04	9.2765E-04	1.4482E 02	1.0519E-02	19
20	3.7815E-02	9.9846E-04	8.6697E-04	1.5244E 02	9.9931E-03	20
21	2.0542E-02	5.8855E-04	7.308E-04	1.6006E 02	9.5172E-03	21
22	-1.8094E-03	9.1640E-04	7.5047E-04	1.6768E 02	9.0846E-03	22
23	3.1829E-03	5.8013E-04	8.5876E-04	1.7530E 02	8.6896E-03	23
24	1.3634E-02	1.3582E-03	1.0034E-03	1.8293E 02	8.3276E-03	24
25	3.0534E-02	7.1698E-04	9.0614E-04	1.9055E 02	7.9945E-03	25
26	5.0755E-02	8.3240E-04	7.4658E-04	1.9817E 02	7.6870E-03	26
27	6.9433E-02	6.0454E-04	6.4316E-04	2.0579E 02	7.4023E-03	27
28	6.5381E-02	5.3117E-04	6.0784E-04	2.1341E 02	7.1379E-03	28
29	7.3639E-02	7.6447E-04	7.4866E-04	2.2104E 02	6.8918E-03	29
30	6.7393E-02	9.3454E-04	8.9174E-04	2.2866E 02	6.6620E-03	30
31	6.3740E-02	9.3342E-04	8.2451E-04	2.3628E 02	6.4471E-03	31
32	5.4789E-02	4.9665E-04	5.9124E-04	2.4390E 02	6.2457E-03	32
33	6.1803E-02	4.3823E-04	4.5776E-04	2.5152E 02	6.0564E-03	33
34	4.2763E-02	4.5795E-04	4.6087E-04	2.5915E 02	5.8783E-03	34
35	2.4726E-02	4.8935E-04	5.1794E-04	2.6677E 02	5.7103E-03	35
36	3.2334E-02	6.3511E-04	5.4344E-04	2.7439E 02	5.5517E-03	36
37	2.0237E-02	3.3778E-04	4.5823E-04	2.8201E 02	5.4017E-03	37
38	-2.1476E-03	5.2223E-04	4.5630E-04	2.8963E 02	5.2595E-03	38
39	-4.9191E-03	4.8294E-04	4.4443E-04	2.9726E 02	5.1246E-03	39
40	5.9307E-03	2.8962E-04	4.0593E-04	3.0488E 02	4.9965E-03	40

TEST FOR PAW SP.DENS=6.4901542E-09

NORMALISED AND STANDARDISED

MINIMUM= -3.42854E 00
 MAXIMUM= 3.36994E 00
 NO OF POINTS REC.= 1000
 NO OF CLASS AMPL.= 40
 MAX.LAG NO= 40
 STD.DEV.CORRECTION= 4.07195E-01

TABLE D-5 AUTOCORRELATION AND SPECTRAL DENSITY FUNCTIONS
COMPUTED FOR EXPERIMENT D

FIGURE D-6
THE AUTOCORRELATION FUNCTION PLOT FOR
EXPERIMENT D

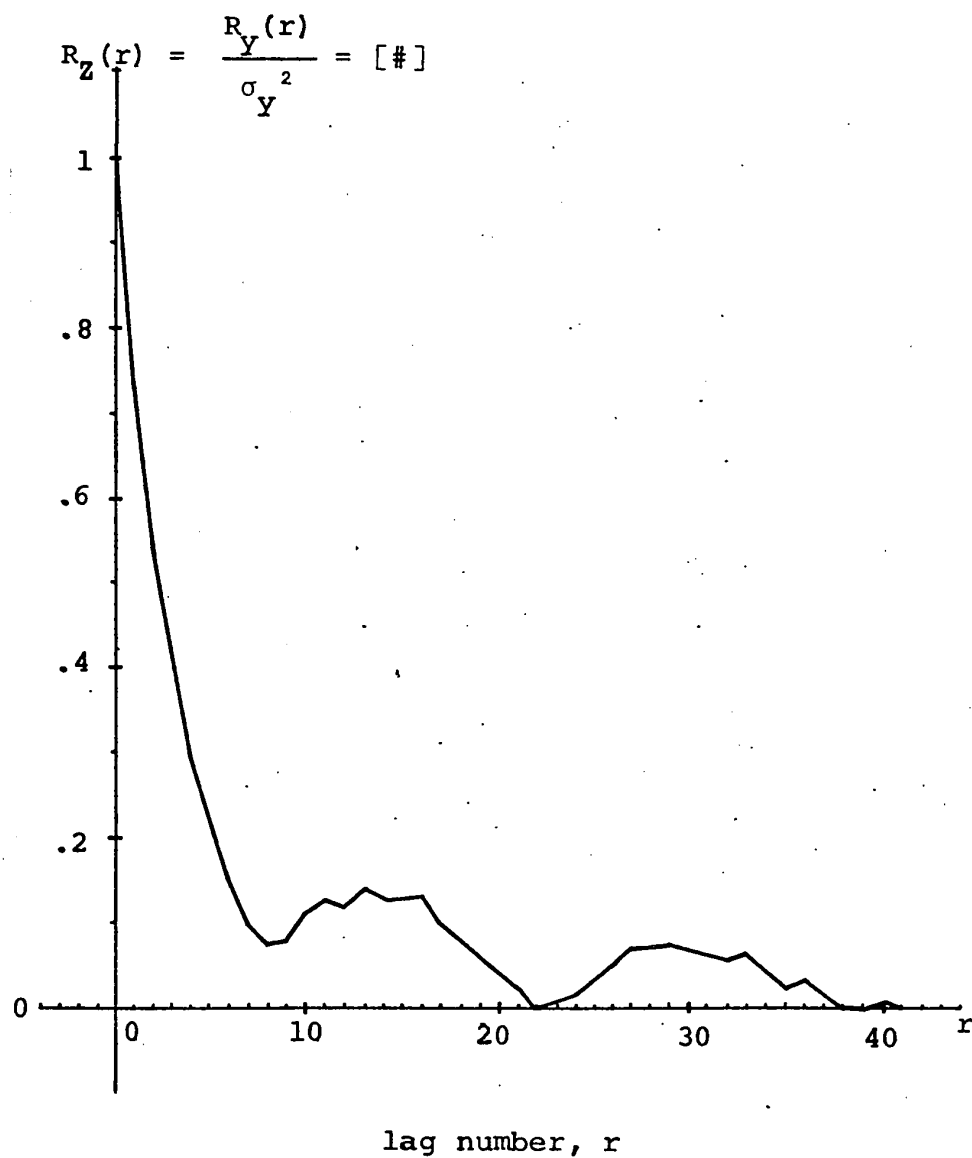


FIG. D-6 THE AUTOCORRELATION FUNCTION PLOT FOR EXPERIMENT D (See also Table D-5)

$$r = 2.5 \mu\text{m}; \quad \sigma_Y = 0.407 \mu\text{m}$$

FIGURE D-7
THE SPECTRAL DENSITY FUNCTION PLOT FOR
EXPERIMENT D

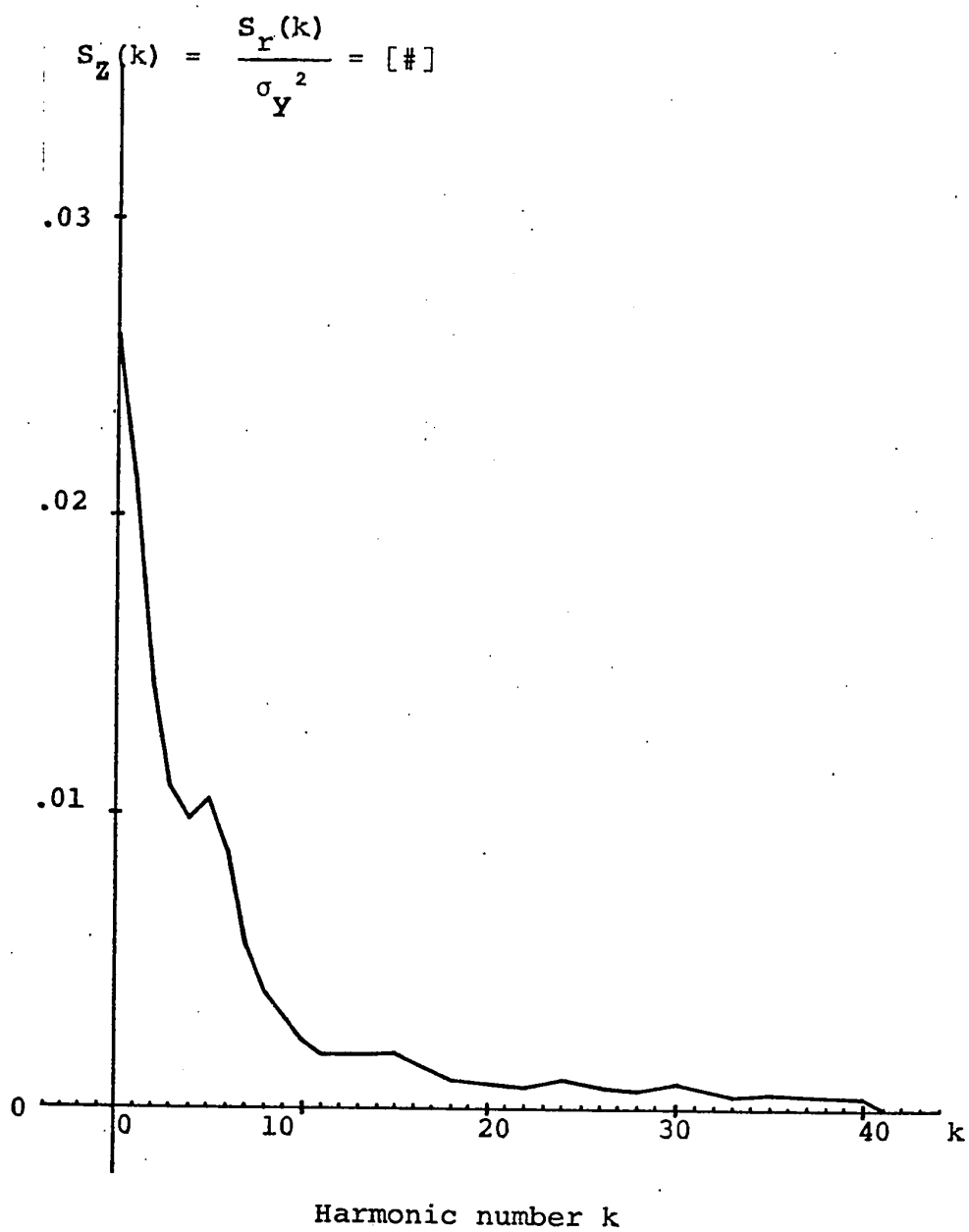


FIG. D-7 THE SPECTRAL DENSITY FUNCTION PLOT FOR EXPERIMENT D (See also Table D-5)

$$\sigma_y = 0.407 \text{ } \mu\text{m}; \quad B_e = 15.25 \text{ cps}$$

APPENDIX E

EXPERIMENT E - RESULTS

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Figure

E-3	The probability density function plot for Experiment E.	E-4
E-4	The probability distribution plot for Experiment E.	E-5
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Table

E-2	The probability density and distribution functions computed for Experiment E	E-3
E-5	Autocorrelation and spectral density functions computed for Experiment E. . .	E-6

E-1 MATERIAL DATA AND PARAMETER SELECTION

E-1 MATERIAL DATA AND PARAMETER SELECTIONMaterial Data

Material	C1020
Manufacturing process	Grinding

Talysurf Set-Up

Sample length	10 mm
Vertical magnification	X 20000
Horizontal magnification	"AV"

Parameters for Data Processing

Number of data points	2000
Number of amplitude class interval	40
Maximum autocorrelation lag number	40
Standardized error for spectral Density	.10

Data has been normalized and standardized

TABLE E-2

THE PROBABILITY DENSITY AND DISTRIBUTION FUNCTIONS

COMPUTED FOR EXPERIMENT E

DISTRIBUTION OF AMPLITUDES.

CLASS	RANGE	MIDPOINT	HISTOGRAM	PERCENT	PROD. DENS.	PROD. DISTR.
0	-INFINITY TO -5.0000	-5.1250	0	0	0	0
1	-5.0000 TO -4.7500	-4.8750	0	0	0	0
2	-4.7500 TO -4.5000	-4.6250	0	0	0	0
3	-4.5000 TO -4.2500	-4.3750	0	0	0	0
4	-4.2500 TO -4.0000	-4.1250	0	0	0	0
5	-4.0000 TO -3.7500	-3.8750	5	.0026	.0104	.0026
6	-3.7500 TO -3.5000	-3.6250	3	.0016	.0062	.0041
7	-3.5000 TO -3.2500	-3.3750	1	.0005	.0021	.0047
8	-3.2500 TO -3.0000	-3.1250	2	.0010	.0041	.0057
9	-3.0000 TO -2.7500	-2.8750	16	.0083	.0332	.0140
10	-2.7500 TO -2.5000	-2.6250	9	.0047	.0167	.0167
11	-2.5000 TO -2.2500	-2.3750	15	.0078	.0311	.0264
12	-2.2500 TO -2.0000	-2.1250	23	.0119	.0477	.0383
13	-2.0000 TO -1.7500	-1.8750	29	.0150	.0601	.0534
14	-1.7500 TO -1.5000	-1.6250	35	.0181	.0725	.0715
15	-1.5000 TO -1.2500	-1.3750	56	.0290	.1161	.1005
16	-1.2500 TO -1.0000	-1.1250	75	.0384	.1554	.1394
17	-1.0000 TO -0.7500	-0.8750	102	.0528	.2114	.1922
18	-0.7500 TO -0.5000	-0.6250	153	.0793	.3171	.2715
19	-0.5000 TO -0.2500	-0.3750	165	.0855	.3420	.3570
20	-0.2500 TO 0	-0.1250	223	.1155	.4622	.4725
21	0 TO .2500	.1250	198	.1026	.4104	.5751
22	.2500 TO .5000	.3750	227	.1176	.4705	.6927
23	.5000 TO .7500	.6250	194	.1005	.4021	.7933
24	.7500 TO 1.0000	.8750	139	.0720	.2881	.8653
25	1.0000 TO 1.2500	1.1250	109	.0565	.2259	.9218
26	1.2500 TO 1.5000	1.3750	55	.0285	.1140	.9503
27	1.5000 TO 1.7500	1.6250	44	.0228	.0912	.9731
28	1.7500 TO 2.0000	1.8750	20	.0104	.0415	.9834
29	2.0000 TO 2.2500	2.1250	14	.0073	.0290	.9907
30	2.2500 TO 2.5000	2.3750	6	.0031	.0124	.9938
31	2.5000 TO 2.7500	2.6250	4	.0021	.0063	.9959
32	2.7500 TO 3.0000	2.8750	3	.0016	.0041	.9974
33	3.0000 TO 3.2500	3.1250	2	.0010	.0021	.9984
34	3.2500 TO 3.5000	3.3750	1	.0005	.0005	.9990
35	3.5000 TO 3.7500	3.6250	1	.0005	.0021	.9995
36	3.7500 TO 4.0000	3.8750	1	.0005	.0021	1.0000
37	4.0000 TO 4.2500	4.1250	0	0	0	1.0000
38	4.2500 TO 4.5000	4.3750	0	0	0	1.0000
39	4.5000 TO 4.7500	4.6250	0	0	0	1.0000
40	4.7500 TO 5.0000	4.8750	0	0	0	1.0000
41	5.0000 TO INFINITY	5.1250	0	0	0	1.0000

MINIMUM= -3.94333E 00
 MAXIMUM= 3.94296E 00
 NO OF CLASS AMPL.= 40
 CLASS INTERVAL SIZE= 2.50000E-01
 MAX LAG NO= 20
 STD DEV CORRECTION= 4.31021E-01

TABLE E-2 THE PROBABILITY DENSITY AND DISTRIBUTION FUNCTIONS COMPUTED FOR EXPERIMENT E

FIGURE E-3
THE PROBABILITY DENSITY FUNCTION PLOT FOR
EXPERIMENT E

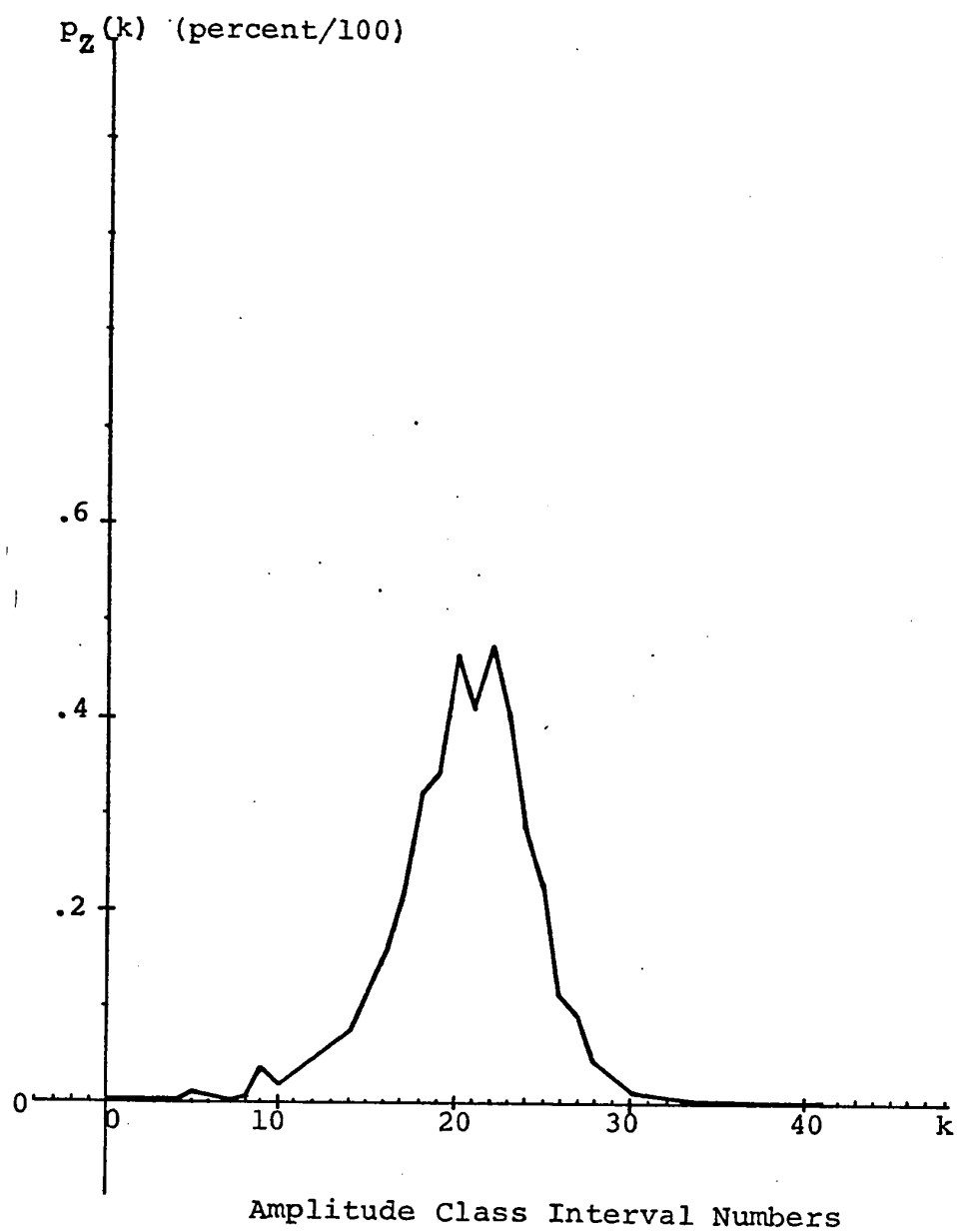


FIG. E-3 THE PROBABILITY DENSITY FUNCTION PLOT
FOR EXPERIMENT E (See also Table E-2)

$$k = 0.25; \quad \sigma_y = 0.431 \mu\text{m}$$

FIGURE E-4
THE PROBABILITY DISTRIBUTION PLOT FOR
EXPERIMENT E

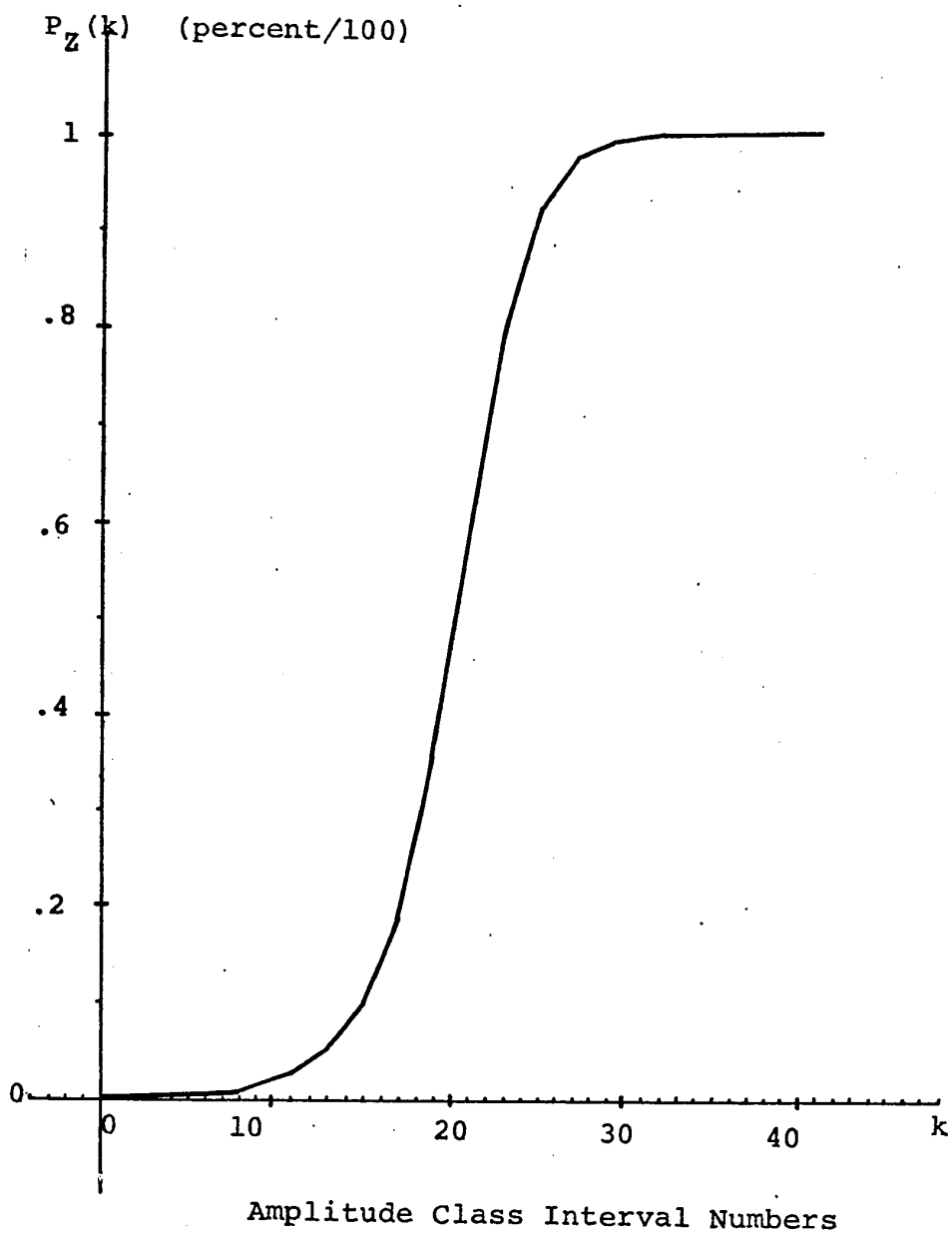


FIG. E-4 THE PROBABILITY DISTRIBUTION PLOT FOR EXPERIMENT E (See also Table E-2)

$$k = 0.25; \quad \sigma_y = 0.431 \mu\text{m}$$

TABLE E-5

AUTOCORRELATION AND SPECTRAL DENSITY FUNCTIONS
COMPUTED FOR EXPERIMENT E

AUTOCORRELATION AND SPECTRAL FUNCTIONS

LAG NO	AUTOCORRELATION	RAW SPECTRAL DENSITY	SMOOTH SPECTRAL DENSITY	FREQUENCY	WAVE LENGTH	HARM.NO.
0	9.9948E-01	4.9742E-02	3.6225E-02	0	0	0
1	5.9798E-01	2.2707E-02	2.7970E-02	7.6220E 00	1.9986E-01	1
2	3.9118E-01	1.6723E-02	1.6402E-02	1.5244E 01	9.9931E-02	2
3	2.9699E-01	9.4583E-03	1.0476E-02	2.2866E 01	6.6620E-02	3
4	2.3168E-01	6.2726E-03	7.6957E-03	3.0488E 01	4.9965E-02	4
5	2.2153E-01	8.7835E-03	7.7197E-03	3.8110E 01	3.9972E-02	5
6	1.8724E-01	7.0340E-03	6.7721E-03	4.5732E 01	3.3310E-02	6
7	1.8480E-01	4.2267E-03	4.9271E-03	5.3354E 01	2.8552E-02	7
8	1.5203E-01	4.2161E-03	4.0490E-03	6.0976E 01	2.4983E-02	8
9	1.0493E-01	3.5370E-03	3.6056E-03	6.8598E 01	2.2207E-02	9
10	7.3020E-02	3.1324E-03	3.2820E-03	7.6220E 01	1.9986E-02	10
11	5.6338E-02	3.3263E-03	2.9670E-03	8.3841E 01	1.8169E-02	11
12	6.9955E-02	2.0829E-03	2.4120E-03	9.1463E 01	1.6655E-02	12
13	6.6385E-02	2.1560E-03	2.2491E-03	9.9085E 01	1.5374E-02	13
14	1.1129E-01	2.6013E-03	2.2719E-03	1.0671E 02	1.4276E-02	14
15	1.1667E-01	1.7292E-03	2.1129E-03	1.1433E 02	1.3324E-02	15
16	9.6243E-02	2.3918E-03	2.2051E-03	1.2195E 02	1.2491E-02	16
17	9.8352E-02	2.3075E-03	2.1262E-03	1.2957E 02	1.1757E-02	17
18	9.3911E-02	1.4940E-03	1.6599E-03	1.3720E 02	1.1103E-02	18
19	8.6272E-02	1.3641E-03	1.4125E-03	1.4482E 02	1.0519E-02	19
20	9.1495E-02	1.4746E-03	1.4888E-03	1.5244E 02	9.9931E-03	20

TEST FOR RAW SP.DENS=1.0477374E-09

NORMALISED AND STANDARDISED

MINIMUM= -3.99333E 00
 MAXIMUM= 3.90298E 00
 NO OF POINTS REC.= 1930
 NO OF CLASS AMPL.= 40
 MAX.LAG NO= 20
 STD.DEV.CORRECTION= 4.31021E-01

TABLE E-5 AUTOCORRELATION AND SPECTRAL DENSITY FUNCTIONS COMPUTED FOR
EXPERIMENT E

SIR GEORGE WILLIAMS UNIVERSITY

COMPUTER CENTER

FIGURE E-6
THE AUTOCORRELATION FUNCTION PLOT FOR
EXPERIMENT E

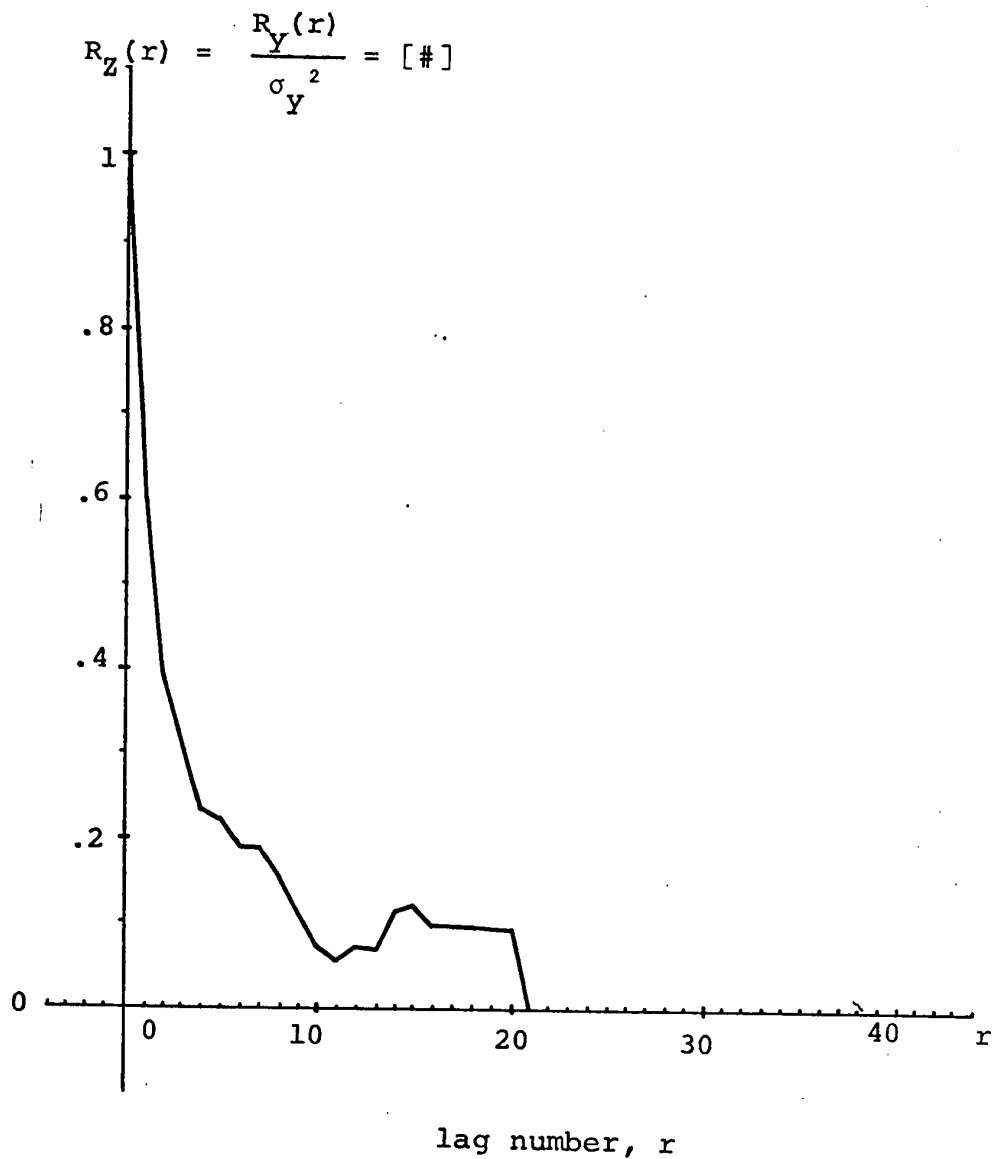


FIG. E-6 THE AUTOCORRELATION FUNCTION PLOT FOR
EXPERIMENT E (See also Table E-5)

$r = 2.5 \mu\text{m};$

$\sigma_Y = 0.431 \mu\text{m}$

FIGURE E-7
THE SPECTRAL DENSITY FUNCTION PLOT FOR
EXPERIMENT E

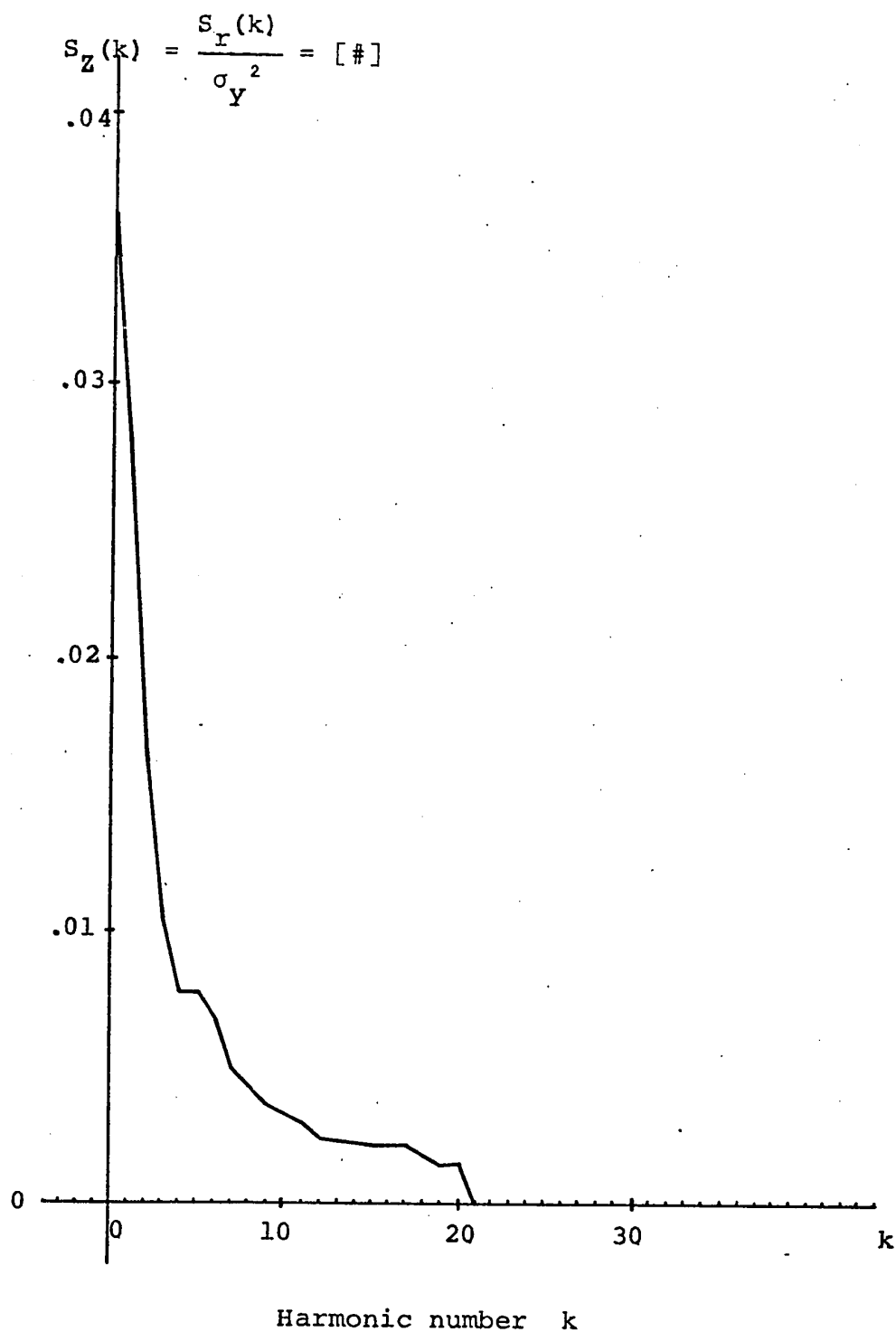


FIG. E-7 THE SPECTRAL DENSITY FUNCTION PLOT FOR
EXPERIMENT E (See also Table E-5)

$$\sigma_Y = 0.431 \mu\text{m}; \quad B_e = 15.25 \text{ cps}$$

END